a place of mind

# FACULTY OF EDUCATION <br> Department of <br> Curriculum and Pedagogy 

## Physics Circular Motion

## Science and Mathematics Education Research Group

## Circular Motion


http://nwsphysics99.blogspot.ca/2013/01/circular-motion.html

https://www.physics.uoguelph.ca/tutorials/shm/phase0.html

## Circular Motion

The following questions have been compiled from a collection of questions submitted on PeerWise (https://peerwise.cs.auckland.ac.nz/) by teacher candidates as part of the EDCP 357 physics methods courses at UBC.

## Circular Motion Problems I

You take a blue ball tied to a string and swing it around your head in circular paths at a constant speed in a counter clockwise direction. If you let go of the string at point P , which direction will the ball follow soon after? Note: The diagram shows a bird's eye view.
A. Direction A
B. Direction B
C. Direction C
D. Direction D


## Solution

## Answer: B

Justification: Remember that an object is said to be moving in uniform circular motion if the object maintains a constant speed (but changing direction) while traveling in a circle. While the object is traveling in a circular path, it is constantly being pulled towards the center, i.e. the centripetal force. A centripetal force is a force that makes an object follow a curved path. The direction is always orthogonal (perpendicular or tangent) to the motion of the object and towards the center of the circle. In our case, when the ball is released at point $P$, we are removing the centripetal force that keeps the ball going in a circular path.

Thus, it will follow a perpendicular direction at P. B is the correct answer.

## Solution continued

## Answer: B

For more information:
https://www.youtube.com/watch?v=Vpyx7Gu0hos
https://www.youtube.com/watch?v=nb4VzvfkSN0
https://en.wikipedia.org/wiki/Circular motion

## Circular Motion Problems II

Suppose the wheels of a Monster Truck have a radius of 1 meter and the wheels of a Smart car have a radius of 20 centimeters. If both vehicles are moving at the same speed, how can we compare the period and frequency of revolution between the wheels of the car and the truck? Let the period and frequency of revolution of the Smart car's wheels be $P_{C}$ and $F_{C}$. Also, let the period and frequency of revolution of the truck's wheels be $P_{T}$ and $F_{T}$.
A. $P_{C}>P_{T}$ and $F_{C}>F_{T}$
B. $P_{C}=P_{T}$ and $F_{C}=F_{T}$
C. $P_{C}<P_{T}$ and $F_{C}<F_{T}$
D. $P_{C}>P_{T}$ and $F_{C}<F_{T}$
E. $P_{C}<P_{T}$ and $F_{C}>F_{T}$

## Solution

## Answer: E

Justification: Note that the period of revolution is the duration to complete one cycle in a repeating event, whereas the frequency of revolution is the number of cycles completed in a given time interval.

Let's think about it! In order to complete one cycle: the truck travels a distance $d=2 \pi r=2 \pi$ meters; the car travels a distance of $d=2 \pi r=2 \pi(0.2)=0.4 \pi$ meters. Since the car and the truck are moving at the same speed, the wheels of the car would have to turn five times faster than the wheels of the truck. Similarly, as the wheels of the car are turning fast within a given period of time, the time required for completing one cycle would be shorter than the wheels of the truck to complete a cycle.

## Solution continued

## Answer: E

This means that the period of the spinning wheels of the car will be less than the truck's spinning wheels.

Thus, $\mathbf{E}$ is the correct answer.
In general, note the inverse relationship between the frequency $(f)$ and the period ( $T$ ). That is, $T=\frac{1}{f}$ or $f=\frac{1}{T}$.

## https://www.youtube.com/watch?v=KuNsQJQ-NJY

## https://en.wikipedia.org/wiki/Frequency

## Circular Motion Problems III

Think of yourself sitting in your physics classroom in Vancouver, which is located at a latitude (or angular distance) of 49 degrees north of the equator. Assuming the radius of the Earth is 6,400 km and the period of Earth's natural rotation is 24 hours, what is your velocity in relation to the natural rotation of the Earth?
A. $233 \mathrm{~m} / \mathrm{s}$
B. $305 \mathrm{~m} / \mathrm{s}$
C. $349 \mathrm{~m} / \mathrm{s}$
D. $465 \mathrm{~m} / \mathrm{s}$


## Circular Motion Problems III (DAVOR)

Think of yourself sitting in your physics classroom in Vancouver, which is located at a latitude (or angular distance) of 49 degrees north of the equator. Assuming the radius of the Earth is 6,400 km and the period of Earth's natural rotation is 24 hours, what is the tangential velocity you experience due to the natural rotation of the Earth?
A. $233 \mathrm{~m} / \mathrm{s}$
B. $305 \mathrm{~m} / \mathrm{s}$
C. $349 \mathrm{~m} / \mathrm{s}$
D. $465 \mathrm{~m} / \mathrm{s}$

http://ourglobalhistory.blogspot.ca/2011/02/how-to-teach-longitude-and-latitude.html

## Solution

## Answer: B

Justification: Remember, the speed of an object moving in a circular path is given by the following equation: $v=\frac{2 \pi r}{T}$, where $r$ represents the radius and $T$ represents the period. Here, it is important to note that $r \neq 6,400 \mathrm{~km}$. Why? One reason is that Vancouver is not located at the equator.
$r$ is the distance from the axis of rotation, which is not the same as the distance from the center of the Earth.

Thus, we need to find $r$.

## Solution continued

Because we're assuming the Earth to be spherical, $r$ can be found using the relation between the sides and angles of a right triangle:
$r=R \sin (\theta)$
Axis of rotation
Where $R$ is the radius of the Earth, and $\theta=90^{\circ}-49^{\circ}=41^{\circ}$

So $r=6400 \sin \left(41^{\circ}\right) \cong 4,200 \mathrm{~km}$


## Solution continued

## Answer: B

Since the period is 24 hours, we can convert from hours to seconds to get $T=24 \times 60 \times 60=86,400 \mathrm{~s}$.

Now, by using $r=4,200 \mathrm{~km}=4,200,000 \mathrm{~m}$ and $T=86,400 \mathrm{~s}$, we can find our speed relative to the natural rotation of the Earth.

Thus, $v=\frac{2 \pi r}{T}=\frac{2 \pi \times 4,200,000}{86,400} \cong 305 \mathrm{~m} / \mathrm{s}$.
Therefore, $\mathbf{B}$ is the correct answer.
http://geography.about.com/video/Latitude-and-Longitude.htm

## Solution continued (DAVOR)

## Answer: B

Since the period is 24 hours, we can convert from hours to seconds to get $T=24 \times 60 \times 60=86,400 \mathrm{~s}$.

Now, by using $r=4,200 \mathrm{~km}=4,200,000 \mathrm{~m}$ and $T=86,400 \mathrm{~s}$, we can find our tangential velocity due to the natural rotation of the Earth.

Thus, $v=\frac{2 \pi r}{T}=\frac{2 \pi \times 4,200,000}{86,400} \cong 305 \mathrm{~m} / \mathrm{s}$.
Therefore, $\mathbf{B}$ is the correct answer.
http://geography.about.com/video/Latitude-and-Longitude.htm

## Circular Motion Problems IV

In continuation of our last problem, what will the effect of the Earth's natural rotation be on the measurement of your weight in Vancouver? Assume: your mass to be 100 kg ; your speed relative to the Earth's natural rotation to be $305 \mathrm{~m} / \mathrm{s}$; and also Vancouver's latitude to be about $49^{\circ} \mathrm{N}$ (or $r=4,200 \mathrm{~km}$ ).
A. Earth's natural rotation will reduce your weight by 2.2 N .
B. Earth's natural rotation will increase your weight by 2.2 N .
C. Earth's natural rotation will reduce your weight by 1.5 N .
D. Earth's natural rotation will increase your weight by 1.5 N .
E. Earth's natural rotation will have no effect on your weight.

## Solution

## Answer: C

Justification: Remember, there are three quantities that will be of interest to us when analyzing objects in circular motion. These three quantities are the speed $(v)$, acceleration $(a)$, and the force $(F)$. They are related to each other through the following relationship: $F=m a$, where $m$ is the mass of the object and $a$ is its acceleration. The acceleration of an object in a circular motion is given by $a=\frac{v^{2}}{r}$.

Since we know that $v=305 \mathrm{~m} / \mathrm{s}, r=4,200 \mathrm{~km}$, and $m=100 \mathrm{~kg}$, we can find the force, $F=m a=m \frac{v^{2}}{r}=100 \times \frac{305^{2}}{4,200,000} \cong 2.2 \mathrm{~N}$.

We are not done yet!

## Solution continued

## Answer: C

$F=2.2 \mathrm{~N}$ is the centrifugal force, which is perpendicular to the axis of the Earth. As a result, we have both vertical and horizontal components to $F$, as shown in the figure below.


## Solution continued

Remember here that when we refer to the vertical and horizontal components, we mean in relation to the point of Earth at which we are at (in Vancouver). We want to know the vertical component because it is this force that will act against Earth's gravity, which is pulling you downward.
In order to find the vertical component of the centrifugal force ( $F_{V}$ ), we can use the relation between the sides and angles of a right triangle. Thus, $F_{V}=F \cos \left(49^{\circ}\right)=2.21 \times \cos \left(49^{\circ}\right) \cong 1.5 \mathrm{~N}$.
Therefore, $\mathbf{C}$ is the correct answer.


## Circular Motion Problems V

Suppose a roller coaster is travelling in a vertical loop of radius of 12 meters. As you travel through the loop upside down, you don't fall out of the roller coaster. What should the minimum speed of the roller coaster be in order for the ride to remain safe at the top of the loop?
A. $6 \mathrm{~m} / \mathrm{s}$
B. $11 \mathrm{~m} / \mathrm{s}$
C. $13 \mathrm{~m} / \mathrm{s}$
D. $18 \mathrm{~m} / \mathrm{s}$

E. $22 \mathrm{~m} / \mathrm{s}$

## Solution

## Answer: B

Justification: The most dangerous position during a roller coaster ride is at the top of the loop. In order to remain safe at the top of the loop, the minimum centripetal force (upward), $F_{C}$, would have to be equal to the force of gravity (the weight), $F_{g}$. That is, $F_{g}=m g=\frac{m v^{2}}{r}=F_{C}$. Simplifying this expression, we get $m g=\frac{m v^{2}}{r} \rightarrow g=\frac{v^{2}}{r} \rightarrow v=\sqrt{r g}$.
Substituting the values for $r$ and $g$, we find $v=\sqrt{r g}=\sqrt{12 \times 10} \cong 11 \mathrm{~m} / \mathrm{s}$. Thus, the minimum speed of the roller coaster has to be around $v=11 \mathrm{~m} / \mathrm{s}$.
Therefore, $\mathbf{B}$ is the correct answer.


## Circular Motion Problems VI

Let's consider the mass and the magnitude of centripetal acceleration of an object in uniform circular motion to be held constant. What will happen if the object's linear/tangential speed is instantly increased?
A. The radius and the object's speed will increase.
B. The radius will increase, but the object's speed will remain constant.
C. The radius will remain constant, but the object's speed will increase.
D. None of the above.

## Solution

## Answer: A

Justification: Holding the acceleration and mass constant implies that the force remains constant. A real life example of this situation could be a hockey player skating in circles. If the hockey player starts skating faster and can't increase the friction between their skates and the ice (here the friction provides the centripetal force), he or she will slide outwards and start traveling around larger circles, at their new speed (the speed they increased to).

Mathematically, centripetal acceleration is given by $a=\frac{v^{2}}{r}$. If we increase $v$ and hold $a$ constant, then $r$ must also increase. An increase in $r$ creates a larger circle.

Therefore, $\mathbf{A}$ is the correct answer.

## Circular Motion Problems VII

Tarzan, the king of the jungle, swings from the top of a cliff holding onto a vine. When Tarzan is at the bottom of the swing, how is the tension force acting on the vine related to the gravitational force of Tarzan?
A. The tension force is greater than the gravitational force.
B. The tension force is equal to the gravitational force.
C. The tension force is less than the gravitational force


## Solution

## Answer: A

Justification: At the bottom of the swing, Tarzan will experience an upward motion due to the centripetal acceleration caused by the tension force in the vine. In order for him to be accelerating upwards at this point along the swing, the tension force pulling Tarzan upwards must be greater than the gravitational force pulling him downwards.

Thus, $\mathbf{A}$ is the correct answer.


