a place of mind

# Physics <br> <br> Electrostatics Problems 

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## Science and Mathematics Education Research Group

## Electrostatics Problems



## Electrostatics Problems

The following questions have been compiled from a collection of questions submitted on PeerWise (https://peerwise.cs.auckland.ac.nz/) by teacher candidates as part of the EDCP 357 physics methods courses at UBC.

## Electrostatics Problems I

You have two uncharged conducting spheres $A$ and $B$ on a nonconducting surface:

After we follow the 4 steps, what are the charges on the spheres?
A. A has a negative charge; $B$ has a positive charge.
B. A has a positive charge; $B$ has a negative charge.
C. A has a negative charge; $B$ has no charge.
D. Both $A$ and $B$ have a negative charge.
E. Both $A$ and $B$ have a positive charge.
$\bigcap_{4}$

## Solution

## Answer: A

Justification: Let us go through the operation step-by-step:

B)Initially, both spheres A and B have no charges. Once these spheres are touching each other they become one conducting object. When a positively charged rod comes close to sphere A without touching it, sphere A becomes negatively charged by induction. This means that the negative charges in both spheres $A$ and $B$ will move towards the positively charged rod (opposite charges attract).
Therefore sphere B becomes positively charged since most of the electrons will have gone to the side of sphere $A$.

## Solution continued

$+1+$
+++


Since they are now oppositely charged, the spheres repel each other.

After the rod is removed, we are left with sphere A, which has a negative charge, and sphere $B$, which has a positive charge. Therefore the correct answer is $\mathbf{A}$.

The method we have used to charge the spheres here is called induction. Induction charging is a method used to charge an object without actually touching the object to any other charged object.

## Electrostatics Problems II

Three balls on strings have been given either a positive charge, a negative charge or no charge. By looking at the diagram below we can conclude that...

A. ...balls 1 and 3 carry charges of opposite sign.
B. ...balls 1 and 3 carry charges of the same sign.
C. ...balls 2 and 3 carry negative charges and ball 1 carries a positive charge.
D. ...balls 2 and 3 carry negative charges and ball 1 carries no charge.
E. None of the above.

## Solution

## Answer: E

Justification: By looking at the diagram, we can see that balls 1 and 2 attract each other and balls 2 and 3 repel each other.

We know that like charges repel each other and opposite charges attract each other. However, we were also told that the balls could also have no charge (i.e. neutral charge).

If an object with a neutral charge is brought close to a charged object, they will always attract each other. This is due to induction - the charged object will induce the electrons within the neutral object to move from one side of the object to the other - see next page for diagram.

## Solution continued

Below are two examples of induction, one with a positively charged object, and another with a negatively charged object. In each case, it is the electrons which move (they are more loosely bound than the protons):

Neutral object
Neutral object


Electrons move towards positively charged object


## Electrons move away

 from negatively charged object
## Solution continued 2

Note that in both cases on the previous page, we did not add or remove any charges from the neutral objects (they still had the same number of positive and negative charges). Instead the neutral objects were polarized - their opposite charges were separated to different sides.

Thus we can see that in the question, since objects 1 and 2 attract each other they may have charges of opposite signs, or one of them could be neutral. We know that 2 and 3 must have charges of equal signs (because they repel), and so 2 cannot be neutral. But we do not know if object 1 is charged or neutral. Therefore we would need to perform further experiments to determine the charges of objects 1,2 and 3.

Thus the best answer is $\mathbf{E}$.

## Electrostatics Problems III

Analyze the following diagram and answer the question on the next page. The absolute value of the positive charge is less than the absolute value of the negative charge. Positions 1, 2 , and 3 are on the same axis as the charges. 1, 2, 3, and 4 are not exact positions on this diagram.

## 4



## Electrostatics Problems III continued

Roughly where could you put a negative point charge so that it doesn't move? The negative point charge is a relatively small charge compared to the absolute values of the two big charges in the previous picture.
A. 1
B. 3
C. 2 and 3
D. 2 and 4
E. 2, 3, and 4

## Solution

## Answer: B

Justification: We need to look at each of the positions that the negative point charge could take:

1) In this case the repulsive force between the negative charge and the negative point charge will dominate the weak attractive force exerted by the positive charge on the negative point charge. Thus, the negative point charge will move to the left.
2) In this case the repulsive force and the attractive force exerted by the 2 big charges will move the negative point charge to the right no matter where you put the point charge between the 2 big charges.

## Solution continued

3) This situation could work. At a certain position, the weak attractive force between the positive charge and the negative point charge (at a short distance) will balance the strong repulsive force between the large negative charge and the negative point charge (at a longer distance) and the point charge will not move.


## Solution continued 2

4) In this case the charge will move to the right due to the repulsion by the negative charge and the attraction of the positive charge.

Therefore, the answer is B representing position 3

## Electrostatics Problems IV

Two charges, 1 and 2, with charges $+q$ and $-q$ respectively are placed a distance rapart as shown in the diagram below. According to Coulomb's Law the magnitude of the force exerted on each charge by the other is:

$$
F_{1 \text { on } 2}=F_{2 \text { on } 1}=F=k \frac{\left|q_{1}\right|\left|q_{2}\right|}{r^{2}}
$$



## Electrostatics Problems IV continued

If the magnitude of $F_{1 \text { on2 }}$ is tripled, what must be true? Select the best answer.
A. $q_{1}=3 q$
B. $q_{2}=-3 q$
C. The magnitude of $F_{2 o n 1}$ is also tripled and its direction is changed.
D. The magnitude of $F_{2 o n 1}$ is also tripled.
E. The magnitude of $F_{2 o n 1}$ is one third of its original value.

## Solution

## Answer: D

Justification: Since the force exerted by each charge is always equal in magnitude, this means that when $F_{10 n 2}$ is tripled, so is $F_{2 o n 1 \text {. If we look at all the possible answers we see that this is the }}$ only answer that must be true:
A) This would cause both $F_{1 o n 2}$ and $F_{2 o n 1}$ to triple, but it does not have to be true because there are other ways to get these forces to triple in magnitude.
B) Same as A - this would cause both $F_{10 n 2}$ and $F_{2 o n 1}$ to triple, but it does not have to be true because there are other ways to get these forces to triple in magnitude.
C) Although the change in magnitude is correct, the change in direction is not necessary.

## Solution continued

D. This must be true if $F_{10 n 2}$ is tripled.
E. This is incorrect because $F_{10 n 2}$ and $F_{20 n 1}$ must be equal

Even though some of the other answers could possibly be true, D is the only answer that must be true in this situation.

## Electrostatics Problems V

Two charges, 1 and 2, with charges $+q$ and $-q$ respectively are placed a distance rapart as shown in the diagram below. According to Coulomb's Law the magnitude of the force exerted on each charge by the other is:

$$
F_{1 \text { on } 2}=F_{2 \text { on } 1}=F=k \frac{\left|q_{1}\right|\left|q_{2}\right|}{r^{2}}
$$



## Electrostatics Problems V continued

What changes can cause this force to be doubled in magnitude?
i. $\quad q_{1}=2 q$
ii. $\quad q_{2}=-\sqrt{2} q$
iii. $r=\frac{1}{2} r$
iv. $q_{2}=-q$ and $r=\frac{1}{\sqrt{2}} r$
v. $q_{1}=\sqrt{2} q$ and $q_{2}=-\sqrt{2} q$

Note: The changes suggested in (i) through (v) do not occur at the same time. For example, if (i) is selected then that change alone should cause the force to double.
A. i and iii
B. ii only
C. iii
D. i, iv and v
E. i and iv

## Solution

## Answer: D

Justification: To begin, the original force exerted by each charge is:

$$
F=k \frac{\left|q_{1}\right|\left|q_{2}\right|}{r^{2}}=k \frac{q q}{r^{2}}=\frac{k q^{2}}{r^{2}}
$$

We know that originally $q_{1}=q$ and $q_{2}=-q$. However, we are only concerned about the magnitude of the force and therefore only the magnitudes of the charges are needed (i.e. signs don't matter).

Next, we want to see what changes in the variables will give a new force, $F_{2}$, which is double the magnitude of $F$. In other words:
$F_{2}=2 F$

## Solution continued

i. $\quad$ Using $q_{1}=2 q$ and $q_{2}=-q$, we get: $F_{2}=k \frac{2 q q}{r^{2}}=2 \frac{k q^{2}}{r^{2}}=2 F$
ii. $\quad U \operatorname{sing} q_{1}=q$ and $q_{2}=-\sqrt{2} q$, we get: $F_{2}=k \frac{\sqrt{2} q q}{r^{2}}=\sqrt{2} \frac{k q^{2}}{r^{2}}=\sqrt{2} F$
iii. Using $r=\frac{1}{2} r$, we get: $F_{2}=k \frac{q q}{\left(\frac{1}{2} r\right)^{2}}=\frac{k q^{2}}{\frac{1}{4} r^{2}}=4 \frac{k q^{2}}{r^{2}}=4 F$
iv. Using $q_{2}=-q$ and $r=\frac{1}{\sqrt{2}} r$, we get: $F_{2}=k \frac{q q}{\left(\frac{1}{\sqrt{2}} r\right)^{2}}=\frac{k q^{2}}{\frac{1}{2} r^{2}}=2 \frac{k q^{2}}{r^{2}}=2 F$
v. Using $q_{1}=\sqrt{2} q$ and $q_{2}=-\sqrt{2} q$, we get: $F_{2}=k \frac{\sqrt{2} q \sqrt{2} q}{r^{2}}=2 \frac{k q^{2}}{r^{2}}=2 F$

From this we see that (i), (iv) and (v) give the answer $F_{2}=2 F$, therefore the answer is $\mathbf{D}$.

## Electrostatics Problems VI

Two charges, 1 and 2, with charges $+q$ and $-q$ respectively are placed a distance rapart as shown in the diagram below. According to Coulomb's Law the magnitude of the force exerted on each charge by the other is:

$$
F_{1 \text { on } 2}=F_{2 \text { on } 1}=F=k \frac{\left|q_{1}\right|\left|q_{2}\right|}{r^{2}}
$$



## Electrostatics Problems VI continued

If $q_{2}=4 q$, how far away do $q_{1}$ and $q_{2}$ need to be such that the force exerted on each charge by the other is one ninth $\left(\frac{1}{9}\right)$ of the original force?
A. $6 r$
B. $\frac{2}{3} r$
C. $16 r$
D. $r$

## Solution

Answer: A
Justification: To do this we need to identify two different situations:

1) The original situation with $q_{1}=+q$ and $q_{2}=-q$ and $r=r$, which has a force of $F$. We know that $F=\frac{k q^{2}}{r^{2}}$
2) The new situation with $q_{1}=+4 q$ and $q_{2}=-q$, which has a force of $F_{\text {new }}=\frac{1}{9} F$ and an unknown distance $r_{\text {new }}$ (we are using $F_{\text {new }}$ and $r_{\text {new }}$ to be able to tell the difference between the two situations).

We can represent the new force, $F_{\text {new }}$ as follows:

$$
F_{\text {new }}=k \frac{4 q q}{r_{\text {new }}{ }^{2}}=\frac{4 k q^{2}}{r_{\text {new }}{ }^{2}} \quad \text { and } \quad F_{\text {new }}=\frac{1}{9} F=\frac{1}{9} \frac{k q^{2}}{r^{2}}
$$

## Solution continued

We can combine the two equations to get:

$$
F_{n e w}=\frac{4 k q^{2}}{r_{n e w^{2}}}=\frac{1}{9} \frac{k q^{2}}{r^{2}}
$$

And then we can solve for $r_{\text {new }}$ :

$$
\begin{aligned}
& \frac{4 k q^{2}}{r_{\text {new }}{ }^{2}}=\frac{1}{9} \frac{k q^{2}}{r^{2}} \\
& \frac{4}{r_{\text {new }}{ }^{2}}=\frac{1}{9 r^{2}} \\
& 36 r^{2}=r_{\text {new }}{ }^{2} \\
& r_{\text {new }}=6 r
\end{aligned}
$$

Therefore the correct answer is $\mathbf{A}$.

## Electrostatics Problems VII

Two point charges of equal mass each experience an acceleration with a magnitude of $a$ due to the electric force between them when they are separated by a distance equal to $r$. What accelerations do they experience when they are separated by a distance of $\frac{r}{2}$ ?
A. $\frac{a}{4}$

Note: Ignore any effects from gravity
B. $\frac{a}{2}$
C. $a$
D. $2 a$
E. $4 a$

## Solution

## Answer: E

Justification: We know that the magnitude of the electric force between the charges can be represented as:

$$
F_{E}=k \frac{\left|q_{1}\right|\left|q_{2}\right|}{r^{2}}
$$

This electric force is equal to the net force of the charges (we are ignoring gravity). Each of the charges experiences an acceleration due to this force, and since the charges have equal mass ( $m$ ), this can be represented as: $\quad F_{E}=m a$
We can see from this equation that the acceleration is directly proportional to the electric force.

## Solution continued

When the two point charges are separated by a distance of $\frac{r}{2}$, we get:

$$
F_{n e w}=k \frac{\left|q_{1}\right|\left|q_{2}\right|}{\left(\frac{r}{2}\right)^{2}}=k \frac{\left|q_{1}\right|\left|q_{2}\right|}{\frac{r^{2}}{4}}=4 k \frac{\left|q_{1}\right|\left|q_{2}\right|}{r^{2}}=4 F_{E}
$$

Since the force $F_{\text {new }}$ has increased to $4 F_{E}$, and the acceleration is proportional to the force, we know that the new acceleration $a_{\text {new }}$ must also have increased four-fold: $a_{\text {new }}=4 a$

Therefore the answer is $\mathbf{E}$

## Electrostatics Problems VIII

An electric field strength created by charge $Q$ is measured to be 40 $\mathrm{N} / \mathrm{C}$ at a distance of 0.2 m from the center of the charge. What is going to be the new field strength if both the magnitude of the charge and the distance from the charge to the point where you measure the field are doubled?
A. $10 \mathrm{~N} / \mathrm{C}$
B. $20 \mathrm{~N} / \mathrm{C}$
C. $40 \mathrm{~N} / \mathrm{C}$
D. $80 \mathrm{~N} / \mathrm{C}$


## Solution

## Answer: B

Justification: Let the electric field strength be denoted by $E$. The magnitude of the electric field strength $(E)$ is defined as the force $(F)$ per charge $(q)$ on the source charge $(Q)$. In other words, $E=\frac{F}{q}$, where $F=\frac{k q Q}{d^{2}}$ is the electric force given by Coulomb's law, k is the Coulomb's law constant ( $k=9.0 \times 10^{9} N \frac{m^{2}}{c^{2}}$ ), and d is the distance between the centers of $q$ and $Q$.

So we need to use the expression, $E=\frac{k q Q}{q d^{2}}$. Simplifying this expression gives, $E=\frac{k Q}{d^{2}}$.

## Solution continued

## Answer: B

In our case, since $Q$ and $d$ are both doubled, the new field strength is $E_{n e w}=\frac{k(2 Q)}{(2 d)^{2}}$, which can be simplified to get $E_{n e w}=\frac{2}{4} \times \frac{k Q}{d^{2}}=\frac{1}{2} E$. Thus, the new field strength is $E_{n e w}=\frac{1}{2} E=\frac{1}{2} \times 40 \mathrm{~N} / \mathrm{C}=20 \mathrm{~N} / \mathrm{C}$.

By doubling the charge from $Q$ to $2 Q$ and the distance from $d$ to $2 d$, our field strength $E$ decreased by half. This happens because of the inverse square law for the distance. E is inversely proportional to distance squared. At the same time it is proportional to the charge:

$$
E \propto \frac{Q}{d^{2}}
$$

## Electrostatics Problems IX

Two point charges $\left(\mathrm{C}_{1}\right.$ and $\left.\mathrm{C}_{2}\right)$ are fixed as shown in the setup below. Now consider a third negative charge $C_{3}$ with charge $-q$ that you can place anywhere you want in regions $A, B, C$, or $D$. In which region could you place charge $\mathrm{C}_{3}$ so that the net force on it is zero?

A. Region A
B. Region B
C. Region C
D. Region D

## Solution

## Answer: D - Somewhere in region D. FIX IT

Justification: With the third charge and $\mathrm{C}_{1}$ being negative, there is a repulsive force on the test charge to the right. From $\mathbf{C}_{2}$, there is an attractive force on the C3 charge to the left. By referring to Coulomb's law ( $F=\frac{k q_{1} q_{2}}{r^{2}}$ ), we know that the force from $\mathrm{C}_{1}$ is being divided by a larger $r$ so that the repulsive force between $\mathrm{C}_{1}$ and the test charge becomes smaller. However, the force from $\mathrm{C}_{2}$ and the test charge is being caused by a smaller magnitude of charge so that the attractive force between $\mathrm{C}_{2}$ and the test charge becomes smaller. At some point in region D, these two effects cancel out and there would be no net force on the test charge.

## Electrostatics Problems X

In each of the four scenarios listed below, the two charges remain fixed in place as shown. Rank the electric potential energies of the four systems from the greatest to the least.
A.

A. $\mathrm{B}=\mathrm{D}>\mathrm{C}>\mathrm{A}$
B. $C>B>A>D$
B. $\bigodot_{3 q} \mathrm{~d} \quad \underset{3 q}{\gtrless}$
C. $\mathrm{C}>\mathrm{B}=\mathrm{D}>\mathrm{A}$
D. $\mathrm{D}>\mathrm{A}=\mathrm{B}>\mathrm{C}$
E. $\mathrm{A}>\mathrm{C}>\mathrm{B}=\mathrm{D}$

D.


## Solution

## Answer: B

Justification: Recall that electric potential energy of a system of electric charges $E_{P}$ depends on two quantities: 1) electric charges involved and 2) the distances between them.

Somewhat similar to the gravitational potential energy of a twocharge system, the electric potential energy is inversely proportional to $r$ and directly proportional to the values of electric charges. The electric potential energy, $E_{P}$, is given by :
$E_{P}=k \frac{q_{1} q_{2}}{r}$, where $k$ is the Coulomb's law constant, $q_{1}$ and $q_{2}$ are the values of point charges, and $r$ is the distance between them.

## Solution continued

## Answer: B

For system A: $E_{P}=k \frac{4 q \times q}{d}=4 k \frac{q^{2}}{d}$
For system $\mathrm{B}: E_{P}=k \frac{3 q \times 3 q}{d}=9 k \frac{q^{2}}{d}$
For system C: $E_{P}=k \frac{2 q \times 10 q}{2 d}=10 k \frac{q^{2}}{d}$
For system D: $E_{P}=k \frac{q \times q}{d / 3}=3 k \frac{q^{2}}{d}$
Since $k \frac{q^{2}}{d}$ is common to all of the above expressions, we note that the numerical coefficients determine the rank of the electric potential energies (i.e., $10>9>4>3$ ). Thus $B$ is the correct answer.

## Electrostatics Problems XI

In each of the four scenarios listed below, the two charges remain fixed in place as shown. Rank the forces acting between the two charges from the greatest to the least.
A.

A. $C>B>A>D$
B. $\mathrm{C}>\mathrm{B}=\mathrm{D}>\mathrm{A}$
C. $\mathrm{B}=\mathrm{D}>\mathrm{C}>\mathrm{A}$
B.

D. $\mathrm{B}+\mathrm{D}>\mathrm{A}>\mathrm{C}$
E. $\mathrm{A}>\mathrm{C}>\mathrm{B}=\mathrm{D}$
C.

D.


## Solution

## Answer: C

Justification: Recall that the electric force is a fundamental force of the universe that exists between all charged particles. For example, the electric force is responsible for chemical bonds. The strength of the electric force between any two charged objects depends on the amount of charge that each object contains and also on the distance between the two charges. From Coulomb's law, we know that the electric force is given by $F=k \frac{q_{1} q_{2}}{r^{2}}$, where $k$ is the Coulomb's law constant, $q_{1}$ and $q_{2}$ are point charges, and $r$ is the distance between the two point charges.

Note that $F$ is proportional to the amount of charge and also inversely proportional to the square of the distance between the charges.

## Solution continued

## Answer: C

For system A: $F=k \frac{4 q \times q}{d^{2}}=4 k \frac{q^{2}}{d^{2}}$
For system $\mathrm{B}: F=k \frac{3 q \times 3 q}{d}=9 k \frac{q^{2}}{d^{2}}$
For system C: $F=k \frac{2 q \times 10 q}{(2 d)^{2}}=5 k \frac{q^{2}}{d^{2}}$
For system D: $F=k \frac{q \times q}{(d / 3)^{2}}=9 k \frac{q^{2}}{d^{2}}$
Since $k \frac{q^{2}}{d^{2}}$ is common to all of the above expressions, we note that the numerical coefficients determine the rank of the electric forces (i.e. $9=9>5>4$ ). Thus C is the correct answer.

## Electrostatics Problems XII

Given the following electric field diagrams:

(a)

(b)

(c)

What are the respective charges of the yellow particles shown in diagrams (a), (b), and (c)?
A. $(a, b, c)=(-q,+q,+q) \quad$ B. $(a, b, c)=(+q, q,-q)$
C. $(a, b, c)=(+q,-q,-2 q)$
D. $(a, b, c)=(-q,+q,+2 q)$
E. $(a, b, c)=(+2 q,-2 q, q)$

## Solution

## Answer: C FIX IT

Justification: Recall that the direction is defined as the direction that a positive test charge would be pushed when placed in the electric field. The electric field direction of a positively charged object is always directed away from the object. And also, the electric field direction of a negatively charged object is directed towards the object.

Direction of am Electric Field


[^0]
## Solution continued

## Answer: C

Since the field direction is directed away from (a) but towards (b) and (c), we know that the relative charges of $(\mathrm{a}, \mathrm{b}, \mathrm{c})=(+,-,-)$

Note that the field lines allow us to not only visualize the direction of the electric field, but also to qualitatively get the magnitude of the field through the density of the field lines. From (a), (b), and (c), we can see that the density of the electric field lines in (c) is twice that of (a) or (b). We would expect the magnitude of the charge in (c) to also be twice as strong as (a) or (b). Thus, the answer choice C is correct.

## Electrostatics Problems XIII

Below is a diagram of a charged object (conductor) at electrostatic equilibrium. Points A, B, and D are on the surface of the object, whereas point $C$ is located inside the object.

Rank the strength of the electric field at points $A, B, C$, and $D$ from strongest to weakest.
A. $\mathrm{B}>\mathrm{D}>\mathrm{A}>\mathrm{C}$
B. $\mathrm{B}>\mathrm{D}>\mathrm{C}>\mathrm{A}$
C. $\mathrm{D}>\mathrm{B}>\mathrm{C}>\mathrm{A}$
D. $\mathrm{D}>\mathrm{B}>\mathrm{A}>\mathrm{C}$
E. $\mathrm{A}>\mathrm{B}>\mathrm{D}>\mathrm{C}$


## Solution

## Answer: D

Justification: We need to understand the concept of the electric field being zero inside of a closed conducting surface of an object, which was demonstrated by Michael Faraday in the 19th century. Suppose to the contrary, if an electric field were to exist below the surface of the conductor, then the electric field would exert a force on electrons present there. This implies that electrons would be in motion. However, the assumption that we made was that for objects at electrostatic equilibrium, charged particles are not in motion. So if charged particles are in motion, then the object is not in electrostatic equilibrium. Thus, if we assume that the conductor is at electrostatic equilibrium, then the net force on the electrons within the conductor is zero. So at point C , the electric field is zero.

## Solution continued

## Answer: D

For conductors at electrostatic equilibrium, the electric fields are strongest at regions along the surface where the object is most curved. The curvature of the surface can range from flat regions to that of being a blunt point, as shown below.

```
Surface Curvature
```



We can notice that the curvature at $D$ is greater than the curvature at B , which, in turn, is greater than the curvature at A . Thus, from the above discussion, we can say that D is the correct answer.


[^0]:    The electric field divection is alwars divected awnay from positive sonnce changer and towands negative sounce changes.

