a place of mind

FACULTY OF EDUCATION

Department of
Curriculum and Pedagogy

Physics
Kinematics Problems

Science and Mathematics
Education Research Group

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Kinematics Problems

Retrieved from: http://www.datamc.org/2013/05/28/accelerometer-data-vs-gps-data/

Kinematics Problems

The following questions have been compiled from a collection of questions submitted on PeerWise (https://peerwise.cs.auckland.ac.nz/) by teacher candidates as part of the EDCP 357 physics methods courses at UBC.

Kinematics Problems I

A car begins driving from a stationary position. It accelerates at 4 m/s^2 for 10 seconds, then travels at a steady speed for another 10 seconds, all in the same direction. How much distance has it covered since it started driving?

A. 200 m

B. 400 m

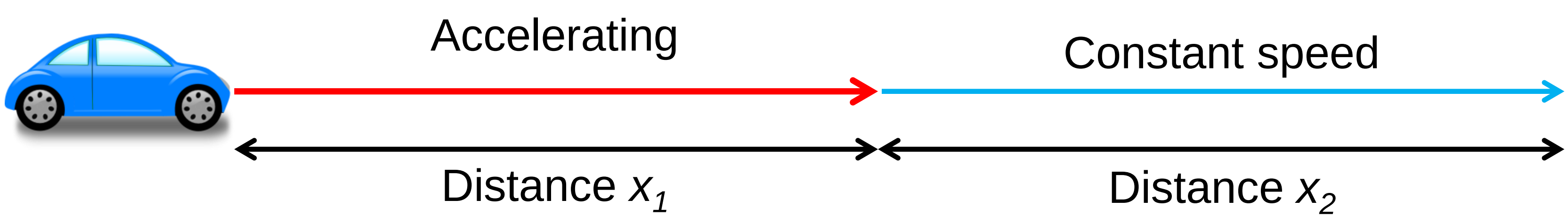
C. 600 m

D. 800 m

Solution

Answer: C

Justification: To answer this question, we need to break it down into two parts, the distance the car traveled while accelerating, and the distance it traveled once it reached its final speed.



To calculate how far it has traveled in the initial ten seconds, we need to use the formula relating acceleration to distance: $x = \boldsymbol{v_i}t + \frac{1}{2}\boldsymbol{a}t^2$

Since the car started at a stationary position, it had an initial velocity ($\boldsymbol{v_i}$) of 0 m/s, and thus we can effectively ignore the first part of the equation.

Solution continued

Therefore:

$$x_i = (0)(10) + \frac{1}{2}(4)(10)^2 = \frac{1}{2}(4)(100) = 200\ m$$

For the second half of the trip, we need to know what speed the car was travelling at the end of its acceleration. We can use the formula relating velocity to acceleration:

$$\boldsymbol{a} = \frac{\Delta \boldsymbol{v}}{\Delta t} = \frac{\boldsymbol{v_f} - \boldsymbol{v_i}}{t_f - t_i}$$

Since the initial velocity and time were both zero, our equation is simplified to:

$$\boldsymbol{a} = \frac{\boldsymbol{v_f}}{t_f}$$

Rearranging it gives us:

$$\boldsymbol{v_f} = \boldsymbol{a}t_f = (4)(10) = 40\ \mathrm{m/s}$$

Solution continued 2

For the second half of the trip, we know that the car travelled for 10 seconds at constant velocity (which we now know was 40 m/s). Therefore we can use the formula relating velocity to distance:

$$x = \boldsymbol{v}t$$
$$x_2 = (40)(10) = 400\ m$$

Adding the two distances together gives us our answer.

$x_{fina} = x_1 + x_2 = 200 + 400 = 600\ m$ (answer **C**)

Kinematics Problems II

Which of the below can be true at the same time?

I. Velocity: Constant; Acceleration: Constant

II. Velocity: Constant; Acceleration: Changing

III. Velocity: Changing; Acceleration: Constant

IV. Velocity: Changing; Acceleration: Changing

A. I only

B. II only

C. IV only

D. III and IV

E. I, III, and IV

Solution

Answer: E

Justification: To answer this question, we will look at each of the options individually:

I – If you have constant velocity, that means that you have zero acceleration. By definition, zero acceleration is constant acceleration. Therefore this option is possible.

II – Similar to above, if you have constant velocity the acceleration must be zero (it cannot be changing). Therefore this option is **not** possible.

III – If you have changing velocity, you will have acceleration. This acceleration CAN be constant (but does not necessarily have to be). The changing velocity can be changing in magnitude, direction or both. Therefore this option is possible.

Solution continued

IV – If you have changing velocity, you will have acceleration. This acceleration CAN be changing (but does not necessarily have to be). If acceleration is changing, $\Delta\boldsymbol{v}$ cannot be constant (this is not high school material). Therefore this option is possible.

Thus we can see that options I, III, and IV are all possible. Therefore the answer is **E**.

Kinematics Problems III

A stone is tossed straight upwards with initial positive velocity of 30 m/s. What is its instantaneous velocity and acceleration at the highest point of its trajectory?

A. Velocity = 0 m/s   Acceleration = 0 m/s^2

B. Velocity = 30 m/s   Acceleration = – 9.8 m/s^2

C. Velocity = – 30 m/s   Acceleration = 0 m/s^2

D. Velocity = 0 m/s   Acceleration = – 9.8 m/s^2

E. Velocity = 30 m/s   Acceleration = 0 m/s^2

Solution

Answer: D

Justification: We can narrow down the answer by looking at what forces are acting on the stone after it is thrown up in the air. Since the only force acting on the stone is the force of gravity, we know that the stone must have a constant downward acceleration of 9.8 m/s^2 (this acceleration does not change during the stone's flight). Since the question has defined upwards as the positive direction, we know that the acceleration experienced by the stone must be – 9.8 m/s^2. Therefore we can eliminate options A, C, and E.

Instantaneous velocity is velocity at a specific time. In this question, it is the velocity at the highest point of the stone's trajectory. At the highest point, the stone is changing its direction from upwards to downwards. Therefore the stone has to stop instantaneously to change its direction completely, therefore its instantaneous velocity would be zero (if it was not zero, the stone would continue to move and this would not be the highest point of the trajectory). Therefore the correct answer is **D**.

Kinematics Problems IV

An object is moving at constant speed. Which statement **MUST** be true?

A. The acceleration of the object must be zero

B. The direction of the object is not changing

C. The velocity of the object is constant

D. All of the above

E. None of the above

Solution

Answer: E

Justification: First we need to make sure that we understand that speed is a **scalar** quantity. This means that it has only magnitude, and no direction. Speed refers to "how fast an object is moving" and can be thought of as the rate at which an object covers distance. Velocity, on the other hand, is a **vector** quantity (has both magnitude and direction) and can be thought of as the rate at which an object changes position.

In our case, if an object experiences constant speed, it is still <u>possible</u> for its position (direction) to be changing (think of a ball swinging in a circle on a rope). Changing direction would result in changing velocity, therefore it is also possible for the velocity to be changing while the object experiences constant speed. With changing direction, it is also possible for the object to be experiencing acceleration (think of the swinging ball from before). Therefore it is possible for the acceleration of the object to be non-zero.

Solution continued

Even though it is possible for the object going at constant speed to also have zero acceleration (A), have no change in direction (B) and also have constant velocity (C), the question asked for which statement **MUST** be true. As we have seen, it is <u>possible</u> for an object going at constant speed to be accelerating, changing in direction and have a changing velocity. Therefore none of the statements can be true, and the correct answer is **E**.

Kinematics Problems V

John stands on the ground and throws a ball directly upwards with a velocity of 5 m/s. What will be the final velocity ***v*** of the ball just before it hits the ground? (Neglect air resistance.)

A. ***v*** **=** 0 m/s

B. ***v*** **=** 5 m/s

C. ***v*** **=** – 5 m/s

D. ***v*** **>** 5 m/s

E. ***v*** **<** – 5 m/s

Solution

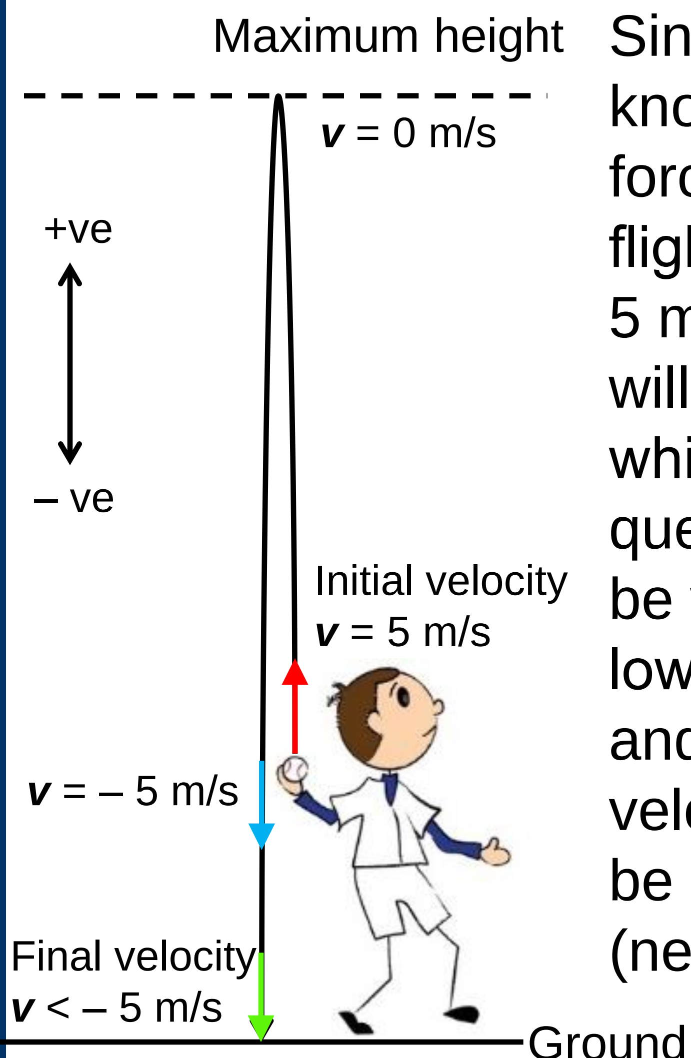
Answer: E

Justification: First we need to figure out the positive and negative axes of the situation. Since we were given the information that the initial velocity of the stone was 5 m/s **upwards**, then this shows us that the upwards direction is positive, and downwards is negative. Since the ball is falling back **down**, we can ignore any answer with positive velocity (answers **B** and **D**).

We also know that the ball will reach 0 m/s when it reaches the maximum height of its trajectory (when the velocity changes from upwards to downwards), so we know it cannot be zero when it is falling down towards the ground. Therefore we can also ignore answer **A**.

The next step is easier to understand if we draw a diagram of the situation (see the next page).

Solution continued



Since we are told to ignore air resistance, we know that the only force acting on the ball is the force of gravity, which is constant throughout its flight. Therefore if the ball leaves John's hand at 5 m/s, we know that when it falls down again it will reach a velocity of – 5 m/s at the height from which it was thrown (John's hand). However, the question asked what the velocity of the ball would be when it hits the **ground**. Since the ground is lower than John's hand, the ball has further to fall and will therefore accelerate more. Thus the velocity it reaches when it hits the ground must be greater than 5 m/s in the **downwards** (negative) direction. Therefore the final velocity must be: ***v* <** – 5 m/s (answer **E**).

Kinematics Problems VI

Roger Federer tosses a tennis ball straight up in the air during his match against Rafael Nadal. If ***a*** is the acceleration of the ball, and ***v*** is its velocity, which statement is true for when the ball reaches the highest point of its trajectory?

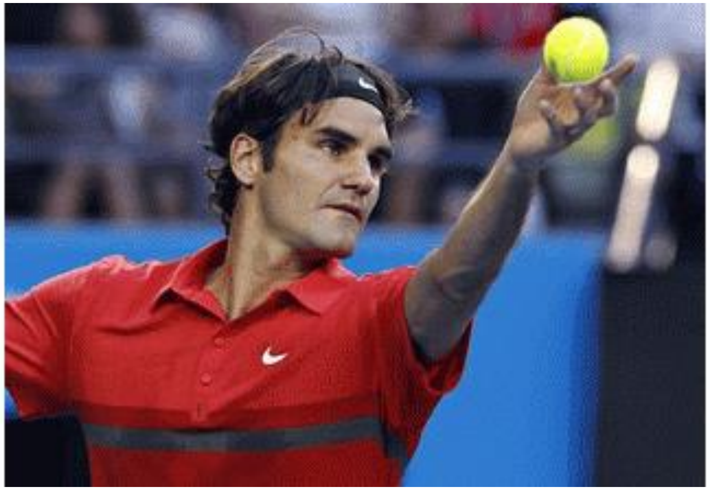
A. Both ***v*** and ***a*** are zero

B. Only ***v*** is zero and ***a*** is not

C. Only ***a*** is zero and ***v*** is not

D. Both ***v*** and ***a*** are non-zero

E. Impossible to answer without knowing the initial speed of the ball



Solution

Answer: B

Justification: At the highest point of the ball's trajectory its velocity must be zero. If it weren't zero, the ball would have continued moving up. This is an instantaneous velocity – the velocity is zero just for a moment as it changes from going up to going down.

The only acceleration the ball is experiencing is the one due to gravity. Since this is constant throughout its flight, the acceleration of the ball cannot be zero at the highest point (if it were, the ball would just stay suspended in the air, since its velocity is zero). The acceleration due to gravity is always acting downwards. Thus after being tossed up the ball's speed is decreasing (it is slowing down), then its speed is momentarily zero, and then it starts speeding up going downwards. It is always under the influence of the gravitational force, so it is accelerating all the time. Thus only answer **B** is correct. Notice, answer E is incorrect, since the initial speed only influences how high up will the ball move before reaching its highest point.

Kinematics Problems VII

When a stone is thrown directly upwards with initial velocity of 30.0 m/s, what will be the maximum height it will reach and when will it be?

Acceleration due to gravity is 10 m/s^2

A. 45 m in 3 s

B. 90 m in 6 s

C. 1.5 m in 3 s

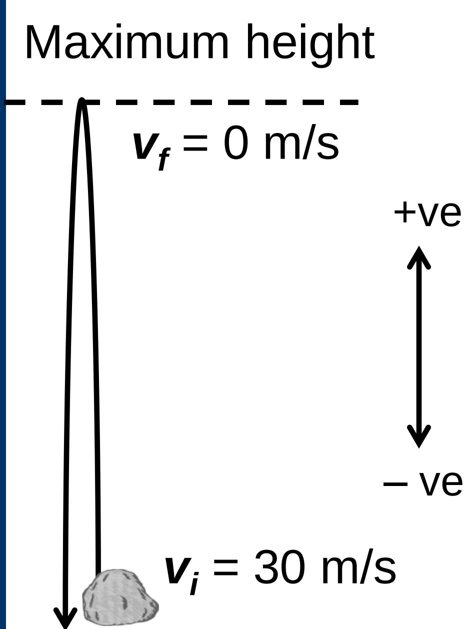
D. 90 m in 3 s

E. 45 m in 6 s

Solution

Answer: A

Justification: First we can draw a diagram and put in our given information. Since we know that initial velocity is positive, then we can take the upwards direction as the positive direction. We also know that since we are looking for the **maximum height** that the stone will reach, at this point the velocity must be 0 m/s.



From the question, the given information is:

Initial velocity ($\boldsymbol{v_i}$) = 30 m/s

Final velocity ($\boldsymbol{v_f}$) = 0 m/s

Acceleration ($\boldsymbol{a}$) = – 10 m/s^2

Since we are looking for the vertical height (d), we should use the following kinematic equation:

$$\boldsymbol{v_f}^2 = \boldsymbol{v_i}^2 + 2\boldsymbol{a}d$$

Solution continued

Therefore we get:

$$\boldsymbol{v_f}^2 = \boldsymbol{v_i}^2 + 2\boldsymbol{a}d$$
$$0 = (30)^2 + 2(-10)d$$
$$20d = 900$$
$$d = 45\ m$$

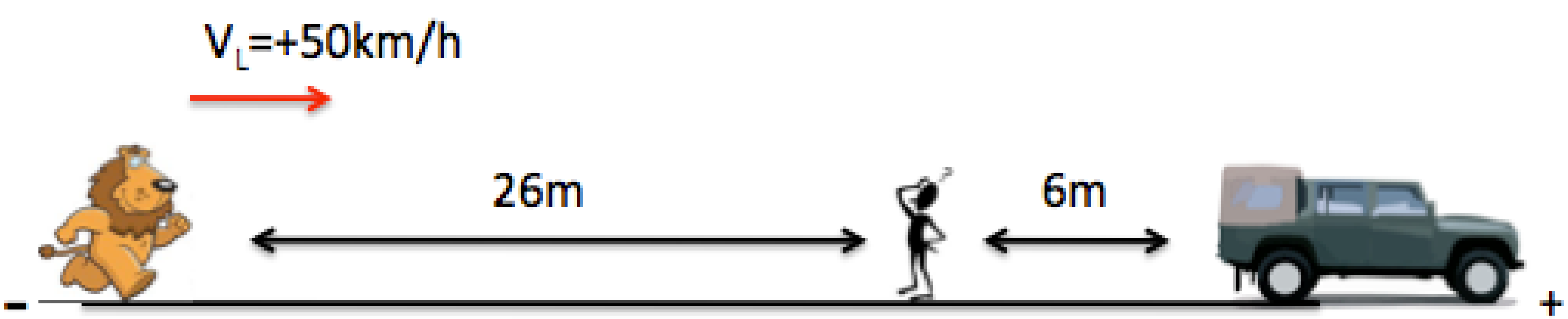
Now we have initial velocity, final velocity, acceleration and height, but not time. Therefore we can choose a kinematic equation which has time in it with any of the other known variables:

$$\boldsymbol{v_f} = \boldsymbol{v_i} + \boldsymbol{a}t$$
$$0 = 30 + (-10)t$$
$$t = 3\ s$$

Therefore the correct answer is **A**.

Kinematics Problems VIII

A lion starts at rest 26 m away from a clueless Jordan and charges towards him at a constant velocity of +50km/h. It takes Jordan 1 s to react to the lion, turn around and begin running at a velocity of +5 m/s towards his vehicle. Jordan's Land Rover is parked 6 m away from him and on the same axis as the lion's charge.



Kinematics Problems VIII continued

If Jordan escapes, how far behind him is the lion?

If Jordan is caught, how far is he from the Land Rover?

A. Jordan escapes and the lion is 1.4 m behind him.

B. Jordan escapes and the lion is 15.3 m behind him.

C. Jordan is caught and is 9.3 m from the Land Rover

D. Jordan is caught and is 6 m from the Land Rover.

E. Jordan is caught and is 1.4 m from the Land Rover.

Solution

Answer: A

Justification: In order to determine if Jordan escapes or is caught we must check if the distance between Jordan and the Lion reaches zero at a time before or after Jordan reaches the Land Rover. The first step in solving this problem is breaking it into two segments. The first is the period in time when the Lion is charging the stationary Jordan (this is the time that it takes Jordan to react), and the second is when Jordan is running back to the Land Rover.

1) Before we can determine how far the lion charges towards Jordan while he is still reacting, we need to convert the speed of the lion from km/h to m/s so that our units do not conflict:

$$\boldsymbol{v_{lion}} = 50 \text{ km/h} = 50 \times \frac{1000}{60} \, m/s = 13.889 \, m/s$$

Solution continued

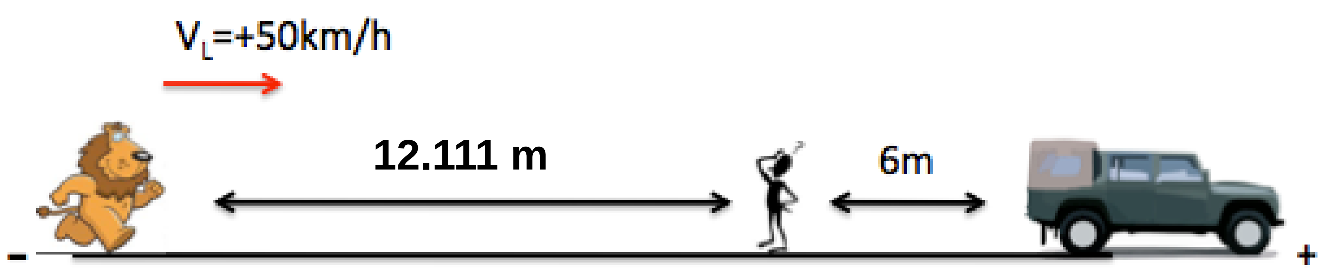
Since we know that it took 1 s for Jordan to react to the lion and that the lion was running at a constant velocity, we can use the following equation to calculate the distance the lion covered initially:

$$d_{initial} = \boldsymbol{v_{lion}} \times t = 13.889 \times 1 = 13.889\ m$$

We know that the lion was initially 26 m away from Jordan, therefore we can now calculate how far away the lion is after the initial 1 s of Jordan reacting:

$$d = 26 - d_{initial} = 26 - 13.889 = 12.111\ m$$

We now have the following situation:



Solution continued

2) In the second part of the problem, Jordan has started running towards his Land Rover at 5 m/s. The lion is still running towards Jordan at 13.889 m/s. Since both Jordan and the lion are travelling at a constant velocity in the same direction, we know that the lion is catching up to Jordan at a constant velocity with a magnitude of the difference between their two velocities:

$$\boldsymbol{v_{relative}} = \boldsymbol{v_{lion}} - \boldsymbol{v_{Jordan}} = 13.889 \ - 5 = 8.889\ m/s$$

What we have done here is looked at the velocity of the lion **relative** to Jordan. From Jordan's frame of reference, it is as if he is standing still, and the lion is moving towards him at a velocity of 8.889 m/s. Looking at it this way makes it much easier for us to calculate at that point the lion will catch up to Jordan. This will happen when the distance between them reaches zero. Since we know the distance between them is currently 12.111 m, we can use the relative velocity to calculate how long it would take for the lion to catch up to Jordan.

Solution continued 2

Therefore:

$$t = \frac{d}{v_{relative}} = \frac{12.111}{8.889} = 1.36\ s$$

So now we know that it would take the lion 1.36 s to catch up to Jordan.

Now we can check to see if Jordan manages to escape or not. We can calculate to see how long it takes for Jordan to run the 6 m distance to the Land Rover:

$$t = \frac{d}{v_{Jordan}} = \frac{6}{5} = 1.2\ s$$

We can now see that it takes less time for Jordan to reach the Land Rover than it takes for the Lion to reach Jordan. Therefore we know that Jordan escapes unscathed.

Solution continued 3

The second part of the question asks how far behind Jordan the Lion is when he reaches the Land Rover. We can answer this by calculating how far the lion runs in the 1.2 s that it takes Jordan to get to his car:

$$d = \boldsymbol{v_{lion}} \times t = 13.889 \times 1.2 = 16.667\ m$$

Since we know that the distance between Jordan and the lion was 12.111 m before he started running, and that the distance between Jordan and the Land Rover was 6 m, then the total distance between the lion and the Land Rover was 18.111 m.

Therefore:

$$d_{final} = 18.111 - 16.667 = 1.444 \approx 1.4\ m$$

Therefore when Jordan reaches his Land Rover, the lion is 1.4 m away from him (answer **A**).

Kinematics Problems IX

Jordan decides to take the bus from Dar Salam to Arusha. The bus leaves Dar at 12:00 pm and drives a distance of 150 km due west in 1.5 h. At this point the driver of the bus realizes he forgot his phone in Dar and must return to the bus station to retrieve it. He takes the same route back and arrives at the bus stop at 3:00pm. What was the average velocity of the bus during the trip?

A. 100 km/h

B. 50 km/h

C. 0 km/h

D. Not enough information

Solution

Answer: C

Justification: For this question we need to remember that velocity is a vector, which means that is has both a magnitude and direction. Since we are looking at average velocity of the trip, we are looking at the change in displacement over time. In other words:

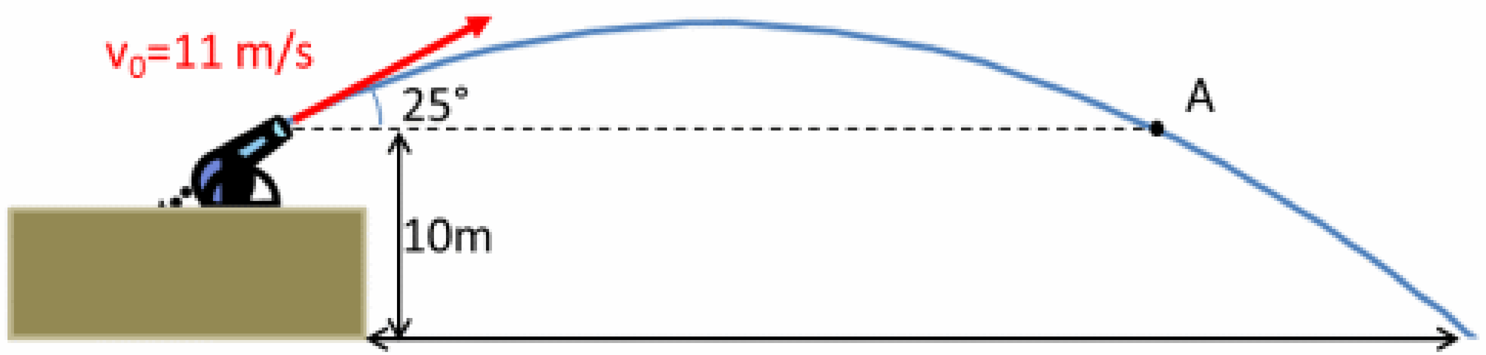
$$v_{average} = \frac{\Delta displacement}{\Delta time}$$

Because the initial location and final location are the same (we started our journey and ended it at the bus stop), the change in displacement will be zero. Therefore the average velocity must be zero (answer **C**).

However, the car does travels a total distance of 300 km in 3 hours, so its average speed is 100 km/h.

Kinematics Problems X

A cannon is firing a ball off a cliff as shown in the image. Find the velocity of the ball when it is passing point A. Neglect air resistance.



A. 4.6 m/s downwards

B. 18 m/s 55 º down from the horizontal

C. 18 m/s 35 º down from the horizontal

D. 11 m/s 25 º down from the horizontal

E. 11 m/s 65 º down from the horizontal

Solution

Answer: D

Justification: You do not need to do any calculations for this question.

In projectile motion, many properties are symmetric with the shape of the parabola (ignoring air resistance). For example, point A is symmetric with the cannon along the trajectory of the ball. The direction of the velocity will be the same 25 º but directed below the horizontal line as opposed to above the horizontal when the cannon ball was fired.

The horizontal (or *x*-component) of the velocity will stay the same throughout the projectile trajectory, but the vertical component (or *y*-component) of the velocity will change depending where the cannon ball is along the trajectory. More specifically, the vertical component will have the same speed but directed at opposite directions (up or down) on symmetrical points on the projectile parabola.

Solution continued

If we look at the velocity at point A again, the same horizontal velocity component with the same vertical velocity component (but directed down instead of up) compared to the starting cannon fire will yield the same final velocity at point A but directed below the horizontal. Therefore the correct answer is 11 m/s 25 º down from the horizontal (**D**)

A) 4.6 m/s = 11 m/s × sin (25 º). This is just the vertical component of the velocity of the ball at point A. The question asks for the velocity, which has both the vertical and horizontal component of the velocity.

B) This is the final velocity at the end of the trajectory when the cannon ball hits the ground.

C) This is the final velocity at the end of the trajectory when the cannon ball hits the ground, but with an incorrect direction.

E) The magnitude of the velocity is correct, but the direction of the velocity is incorrect.

Kinematics Problems XI

A 1 kg ball is fired from a cannon directly upwards in an airless chamber. Its initial speed is 10 m/s, and it reaches a height of 5 m before falling due to Earth's gravity. If a 2 kg ball is fired at 20 m/s directly upwards, what height will it reach before it begins falling, rounded to the nearest meter?

Acceleration due to gravity is 10 m/s^2.

A. 3m

B. 5m

C. 10m

D. 20m

Solution

Answer: D

Justification: We know that the acceleration due to gravity is uniform regardless of an object's mass, so the change in mass between the two balls has no effect on the distance the ball will travel upwards. In addition, because the cannon is firing in an airless chamber, we don't need to worry about air resistance. We also know that once the ball reaches the highest point of its journey, its speed will be 0 m/s before it starts falling again, and this is true regardless of what its initial speed was. We can use the equation that equates an object's acceleration, initial and final speed, and the distance it travels, to determine the answer:

$$\boldsymbol{a} = \frac{(\boldsymbol{v_f}^2 - \boldsymbol{v_i}^2)}{2\boldsymbol{s}} \quad \rightarrow \quad \boldsymbol{s} = \frac{(\boldsymbol{v_f}^2 - \boldsymbol{v_i}^2)}{2\boldsymbol{a}}$$

Solution continued

We know that:

$\boldsymbol{a}$ = – 10 m/s^2 (We add a negative sign to the acceleration because it is acting in the opposite direction compared to the direction of the ball's speed.)

$\boldsymbol{v_f}$ = 0 m/s

$\boldsymbol{v_i}$ = 20 m/s

Therefore:

$$\boldsymbol{s} = \frac{((0)^2 - (20)^2)}{2(-10)} = \frac{-400}{-20} = 20\ m/s$$ (answer **D**)

Solution continued 2

Answer A is not correct, because we know that the higher initial speed will increase the height that the ball travels, not decrease it.

Answer B is not correct, because even though the acceleration due to gravity is the same on both balls, the increased initial speed for the second ball means that it will take more time for gravity to slow it down. It will exceed the height of the initial ball.

Answer C is not correct. The doubled mass will have no effect on the height of the ball's trajectory. Also, if you doubled the initial velocity, we know that the relationship between acceleration, initial and final speeds, and distance travelled involves the square of the speed ($\boldsymbol{v_i^2}$)***,*** so the distance travelled will be increased not by a factor of 2, but by a factor of 2^2 (a factor of 4).

Kinematics Problems XII

A curling stone is moving along a frictionless curling sheet and is being pushed by a constant horizontal force from a player. The stone must be...

A. ...moving at a constant velocity

B. ...moving at a constant speed

C. ...moving at a constant acceleration

D. ...moving at an increasing acceleration

Solution

Answer: C

Justification: The curling stone has a net constant horizontal force. No force of friction is applied. Since $\boldsymbol{F} = m\boldsymbol{a}$ according to Newton's 2nd law, the stone must be moving at a constant acceleration (the mass of the stone does not change). Therefore the correct answer is **C**.

A. Constant velocity would mean that there is no acceleration.

B. Constant speed means that the magnitude of acceleration is zero.

D. Increasing acceleration is not possible since the horizontal force is constant.

Kinematics Problems XIII

Jordan's flight from Zurich to Vancouver can be approximated by a 8,302 km vector due west. The jet that Jordan is flying on has a top speed of 700 km/h and there is a 80 km/h wind blowing in the direction of 60 degrees south of east.

What is the shortest amount of time Jordan's flight can take?

A. 12 hr 38 min

B. 11 hr 52 min

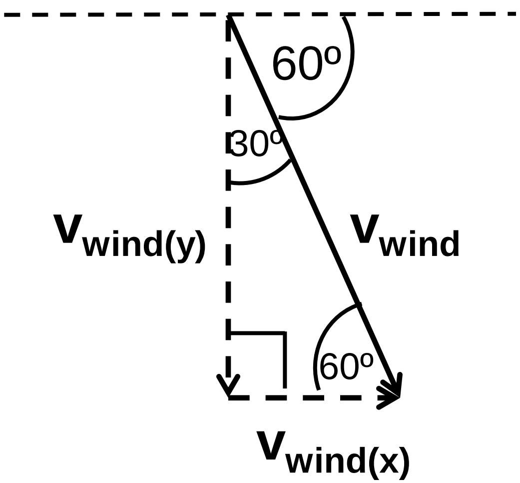
C. 14 hr 4 min

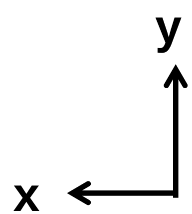
D. 13 hr 23 min

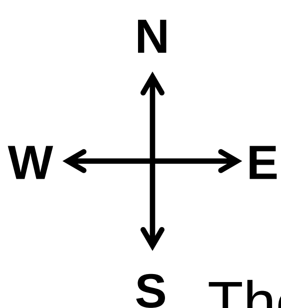
Solution

Answer: A

Justification: The first step is to break the velocity of the wind ($\mathbf{v}_{\mathbf{wind}}$) into its components. Here we shall define the West-East axis to be the x-axis, and the North-South axis to be the y-axis.







We can use the Sine rule to find out the values of the x and y components of $\mathbf{v}_{\mathbf{wind}}$:

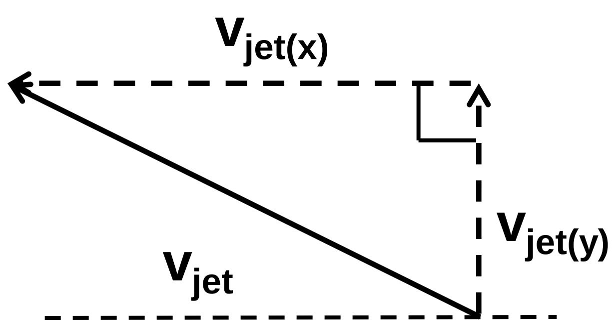
$$\mathbf{v}_{\mathbf{wind(x)}} = -\ 80\ \sin(30^{\circ}) = 40\ \text{km/h}$$

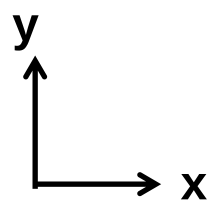
$$\mathbf{v}_{\mathbf{wind(y)}} = -\ 80\ \sin(60^{\circ}) = 69.3\ \text{km/h}$$

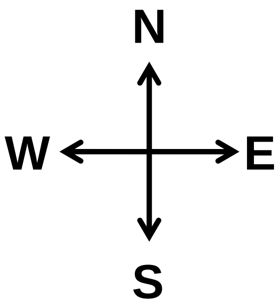
The speeds are **negative** because we have defined West as the positive x direction and North as the positive y direction

Solution continued

In order for the jet to stay on its westward course (and not be blown towards the South), it will need to have a Y component to its velocity that is equal to the Y component of the wind's velocity (69.3 km/h). Since we know the top speed of the jet ($\mathbf{v}_{jet}$ = 700 km/h), we can draw a vector diagram of the jet's flight:







We can use the Theorem of Pythagoras to find the x component of $\mathbf{v}_{jet}$:

$\mathbf{v}_{jet(x)}{}^2 = \mathbf{v}_{jet}{}^2 – \mathbf{v}_{jet(y)}{}^2$

$\mathbf{v}_{jet(x)}{}^2 = (700)^2 – (69.3)^2$

$\mathbf{v}_{jet(x)}{}^2 = 490\,000 – 4800$

$\mathbf{v}_{jet(x)} = 696.6$ km/h

Solution continued 2

Now that we know the top velocity of the jet in the westward direction ($\mathbf{v}_{\mathbf{jet(x)}}$ = 696.6 km/h), we can subtract the eastern (x) component of the wind ($\mathbf{v}_{\mathbf{wind(x)}}$ = 40 km/h) to find the net velocity of the jet relative to the ground:

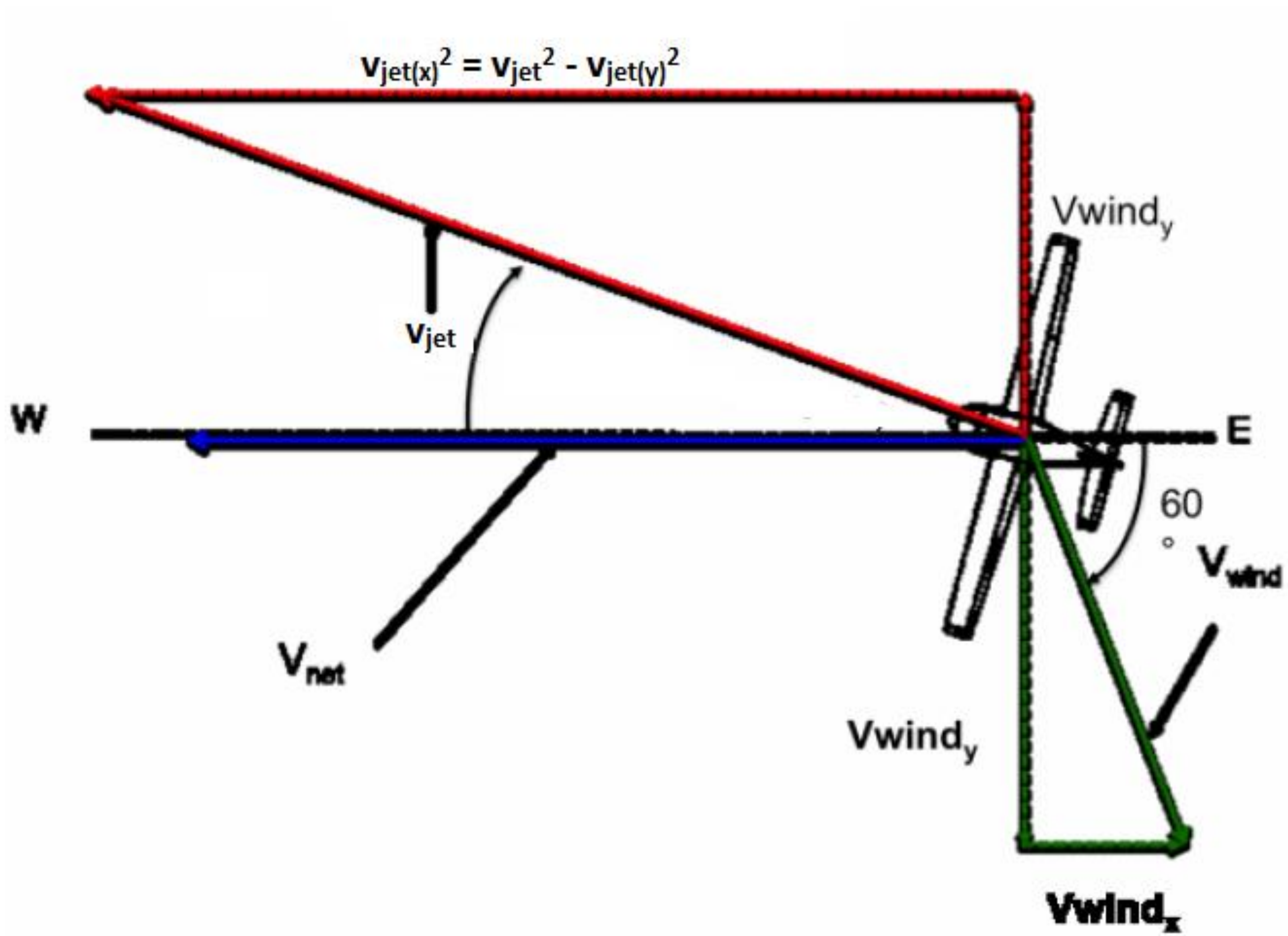
$\mathbf{v}_{\mathbf{net}} = \mathbf{v}_{\mathbf{jet(x)}} - \mathbf{v}_{\mathbf{wind(x)}} = 696.6 - 40 = 656.6$ km/h

Then we divide the total distance by the velocity relative to the ground to find the time it takes the plane to make the trip.

Time = Distance/Velocity = 8302/656.6 = 12.64 hours

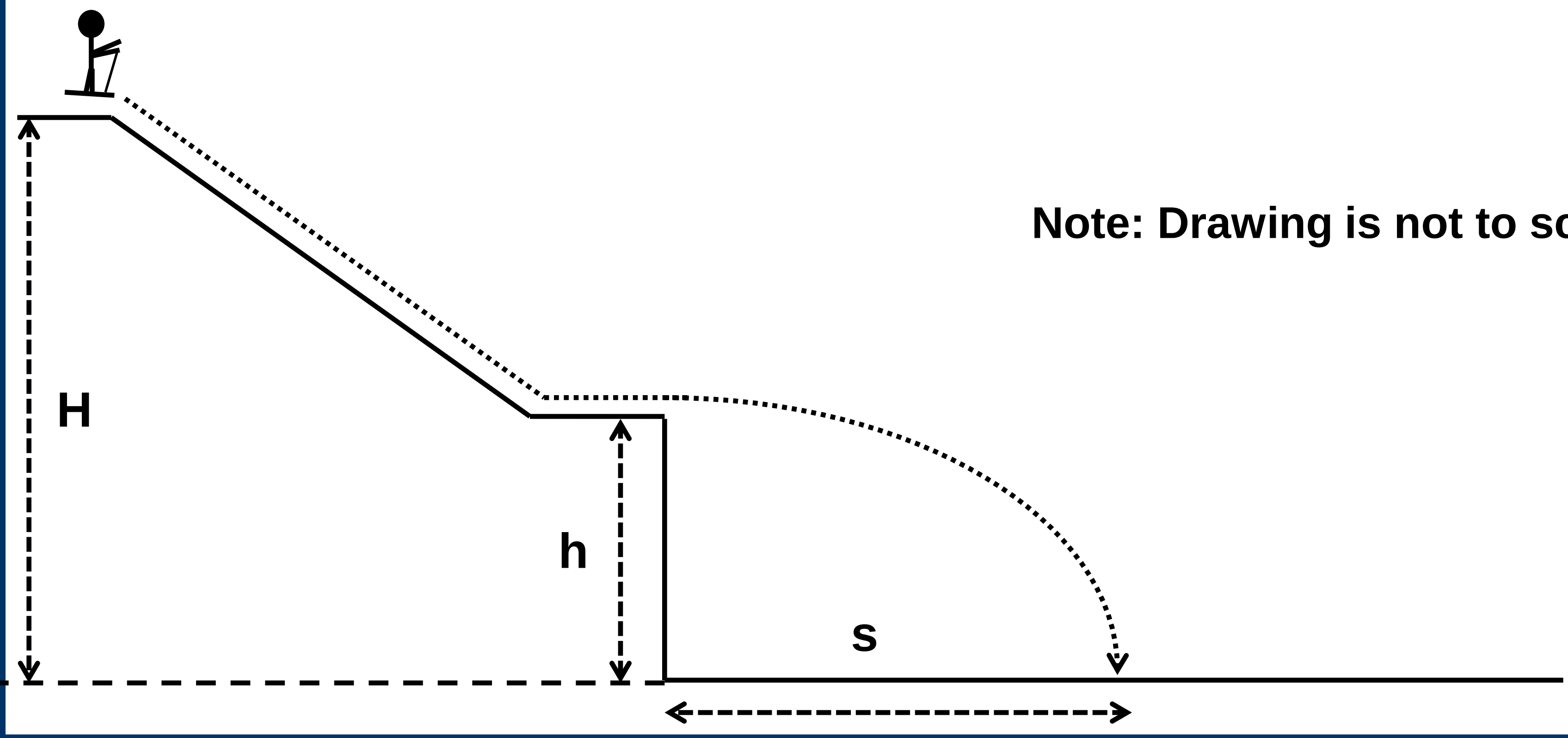
We can convert the decimal hours to minutes (×60) to get the final answer of 12 hours and 38 minutes (Answer **A**)

Solution continued 3



Kinematics Problems XIV

A ski slope begins on a hill **H** meters high and ends in a short horizontal lip before dropping **h** meters to the ground in a sheer vertical cliff. A slalom skier, starting from rest, skies down this slope to land on level ground, **s** meters away from the cliff face.



Kinematics Problems XIV continued

What is the maximum possible value of **s**? Assume that friction on the slope is negligible, ignore air resistance.

A. Zero

B. H/4

C. H/2

D. H

E. 2H

Solution

Answer: D

Justification: Since the skier starts from rest, we know that at the top of the hill they only have potential energy: $PE_{top} = m\mathbf{g}H$

Once the skier reaches the cliff edge, their potential energy has been reduced to $PE_{midway} = m\mathbf{g}h$

We can use the difference between these two potential energies to find out how much kinetic energy the skier gained by going down the slope:

$$KE_{midway} = PE_{top} - PE_{midway} = m\mathbf{g}H - m\mathbf{g}h = m\mathbf{g}(H - h)$$

We can use the equation for kinetic energy to work out the velocity of the skier just before they launch off the cliff edge:

$$KE_{midway} = \tfrac{1}{2} m\mathbf{v}^2 = m\mathbf{g}(H - h)$$

$$\mathbf{v}^2 = 2\mathbf{g}(H - h)$$

Therefore $\mathbf{v} = \sqrt{2g(H - h)}$

Solution continued

Since the skier launches off horizontally, they effectively launch with a **constant horizontal** velocity of $\sqrt{2g(H-h)}$, and with an **initial vertical** velocity of zero.

We can calculate the time, t, that it would take the skier to fall to the ground. This value only relies on the acceleration due to gravity (**g**) and his initial **vertical** velocity(which is zero). We can use the following two equations:

$\mathbf{v_f} = \mathbf{v_i} + \mathbf{a}t$ and $\mathbf{v_f}^2 = \mathbf{v_i}^2 + 2\mathbf{a}d$

$\mathbf{v_f} = (0) + (\mathbf{g})t = \mathbf{g}t$ $\mathbf{v_f}^2 = (0)^2 + 2(\mathbf{g})(h) = 2\mathbf{g}h$

Then we substitute $\mathbf{v_f}$: $(\mathbf{g}t)^2 = 2\mathbf{g}h$

$$t^2 = \frac{2h}{g} \rightarrow t = \sqrt{\frac{2h}{g}}$$

Solution continued 2

Now we can use this time (the time it takes the skier to fall down to the ground) to calculate the **horizontal** distance they cover (s in the diagram). Since they have a **constant** horizontal velocity of $\sqrt{2g(H-h)}$, we can simply use the equation:

Distance = Velocity × Time

$$s = \sqrt{2g(H-h)} \times \sqrt{\frac{2h}{g}} = \sqrt{4h(H-h)} = 2\sqrt{h(H-h)}$$

To maximize s with respect to h, consider maximizing s^2 with respect to h. In this case the equation would be $s^2 = 4h(H - h)$.

The plot of s^2 versus h is a downward opening parabola with its vertex at $h = H/2$ and $s^2 = H^2$ which yields the maximum value of H for s (answer **D**).

Kinematics Problems XV

A 5kg box is pushed and then let go. It moves in the right direction along a horizontal surface with a kinetic coefficient of friction $\mu = 0.3$. What is the acceleration of the box?

Note: The right direction is the positive direction.

Acceleration due to gravity $g = 10\ m/s^2$

A. $3\ m/s^2$

B. $0.3\ m/s^2$

C. $-0.3\ m/s^2$

D. $-3\ m/s^2$

E. $-15\ m/s^2$

Solution

Answer: D

Justification: Since there is nothing pushing on the box, the only force acting on the box is the force of friction.

We can calculate the friction force with the equation: $\mathbf{F_f} = \mu \times \mathbf{F_N}$

$\mathbf{F_N}$ is the normal force, and since there is no vertical acceleration, the normal force is equal to the force of gravity on the box (F_g).

$\mathbf{F_g} = m\mathbf{g}$

Therefore $\mathbf{F_f} = \mu \times \mathbf{F_g} = \mu \times m\mathbf{g} = (0.3) \times (5) \times (10) = 15\ \mathrm{N}$

We know that the friction force acts **opposite** to the direction of motion of the box (i.e. to the left), therefore it must be **negative**. So $\mathbf{F_f} = -15\ \mathrm{N}$

Since the force of friction is the only force acting on the box, we can say:

$\mathbf{F_f} = \mathbf{F_{net}} = m\mathbf{a}$

So $(-15) = (5)\mathbf{a}$, and therefore $\mathbf{a} = -3\ \mathrm{m/s^2}$ (answer **D**)

Kinematics Problems XVI

A 45 kg skier is going at +8.2 m/s down an icy ski run when she suddenly falls and starts sliding on her back (in the same direction she was going). Exactly 3.0 s after falling her velocity has changed to +3.1 m/s.

How far has she slid down the hill from where she fell?

Assume that the acceleration is constant.

A. 16.95 m

B. 19.95 m

C. 68.75 m

D. 2.94 m

Solution

Answer: A

Justification: The initial velocity $\mathbf{v_i}$, final velocity $\mathbf{v_f}$, and the change in time Δt have been given in the question. In order to figure out how far the skier falls in the given time we need to use the kinematics formula that has distance **d** in it. The following equation can be used to find **d**:

$$\boldsymbol{d} = \frac{\boldsymbol{v_i} + \boldsymbol{v_f}}{2} \times \Delta t$$

Where $\mathbf{v_i}$ = 8.2 m/s, $\mathbf{v_f}$ = 3.1 m/s, Δt = 3.0 s

Therefore: $\boldsymbol{d} = \frac{8.2 + 3.1}{2} \times 3.0 = 16.95\, m$ (answer **A**)

Notice here that we did not have to make use of the skiers mass (45 kg) to solve the problem.

Kinematics Problems XVII

A 45 kg skier is going at +8.2 m/s down an icy ski run when she suddenly falls and starts sliding on her back (in the same direction she was going). Exactly 3.0 s after falling her velocity has changed to +3.1 m/s.

What is the skiers acceleration as she slides down the hill?

Assume that the acceleration is constant.

A. 9.81 m/s^2

B. 1.7 m/s^2

C. –1.7 m/s^2

D. 9.61 m/s^2

E. – 9.81 m/s^2

Solution

Answer: C

Justification: To start it is imperative to understand that the skier is not in free fall. Additionally it is important to recognize that the initial velocity $\mathbf{v_i}$, final velocity $\mathbf{v_f}$, and change in time Δt are provided in the question. This means that the following kinematic equation can be used:

$$\boldsymbol{v_f} = \boldsymbol{v_i} + \boldsymbol{a}\Delta t$$

Where $\mathbf{v_i}$ = 8.2 m/s, $\mathbf{v_f}$ = 3.1 m/s, Δt = 3.0 s

We can rearrange the equation to get:

$$\boldsymbol{a} = \frac{\boldsymbol{v_f} - \boldsymbol{v_i}}{\Delta t} = \frac{3.1 - 8.2}{3.0} = -1.7\, m/s^2$$ (answer **C**)

Note: it is possible to use a combination of other kinematic equations to solve this problem (there is no "one way" to solve it). For example the equation $\boldsymbol{d} = \frac{\boldsymbol{v_i} + \boldsymbol{v_f}}{2} \times \Delta t$ can be solved for **d** (previous question). Once **d** has been found, any of the other kinematic equations that have "**a**" and "**d**" as variables can be used to solve for **a**.

Solution continued

This is a breakdown of why the other options were incorrect:

A. **a** = 9.81 m/s^2 is incorrect because this is assuming that the skier is free falling and has an acceleration of that due to gravity on earth. It is also incorrect because the acceleration is greater than zero which implies that the skier is speeding up as she falls, not slowing down as stated by the question.

B. **a** = 1.7 m/s^2 is incorrect because this answer implies that the skier speeds up after falling. Although the correct equation was used (as shown above) it is likely that the values for $\mathbf{v}_i$ and $\mathbf{v}_f$ were swapped or a negative sign was forgotten in the final answer.

D. **a** = 9.605 m/s^2 is incorrect because it uses the equation $\mathbf{v}_f^2 = \mathbf{v}_i^2 + 2\mathbf{ad}$ with **d** = 3.0. This equation should not be used for this question because no distances are mentioned in the question.

E. **a** = 9.81 m/s^2 is incorrect because this is assuming that the skier's acceleration has the same magnitude as that of the acceleration due to gravity, but this does not properly take into account the information provided in the question.

Kinematics Problems XVIII

A 45 kg skier is going at +8.2 m/s down an icy ski run when she suddenly falls and starts sliding on her back (in the same direction she was going). Exactly 3.0 s after falling her velocity has changed to +3.1 m/s.

How long will it take her to come to a complete rest from the time she began falling?

Assume that the acceleration is constant.

A. 3.0 s

B. 3.3 s

C. 1.8 s

D. 4.8 s

Solution

Answer: D

Justification: This question can be solved in many different ways. Here will be shown two different methods that can be used.

METHOD 1:

Here, let us say that there are two important points in time:

1. the moment the skier started falling (the initial time), and
2. the moment the skier came to a complete rest (the final time).

From the previous question we know that the skier's acceleration is $\mathbf{a} = -1.7\ \text{m/s}^2$ while she is falling. We also know that her velocity when she began falling is $\mathbf{v_i} = 8.2$ m/s. Finally, we know that when any object comes to a complete rest its velocity is zero, therefore we can say that $\mathbf{v_f} = 0$ m/s. With this information the following equation can be used and solved for Δt:

$$\mathbf{v_f} = \mathbf{v_i} + \mathbf{a}\Delta t$$

Solution continued

Rearranging the equation and putting in the values in for $\mathbf{v_f}$, $\mathbf{v_i}$ and $\mathbf{a}$ we get:

$$\Delta t = (\mathbf{v_f} - \mathbf{v_i})/\mathbf{a}$$

$$\Delta t = (0 - 8.2)/(-1.7)$$

$$\Delta t = 4.8 \text{ s} \quad \text{(answer } \mathbf{D}\text{)}$$

METHOD 2:

Here, let us say that there are two important points in time:

1.exactly 3.0 s after the moment the skier starts falling (the initial time) and

2.the moment the skier came to a complete rest (the final time).

From the previous question we know that the skier's acceleration is $\mathbf{a} = -1.7 \text{ m/s}^2$ while she is falling. We also know that exactly 3.0 s after she started falling her velocity is $\mathbf{v_i} = 3.1$ m/s. Finally, we know that when any object comes to a complete rest its velocity is zero, therefore we

Solution continued 2

can say $\mathbf{v_f}$ = 0 m/s. With this information the following equation can be used and solved for Δt:

$$\mathbf{v_f} = \mathbf{v_i} + \mathbf{a}\Delta t$$

NOTE: In this case Δt is the time from when the skier is moving at exactly 3.1 m/s to when she comes to a complete rest. It does NOT include the 3.0 s after she began to fall (we will add this on later).

Rearranging the equation and putting in the values in for $\mathbf{v_f}$, $\mathbf{v_i}$ and **a** we get:

$$\Delta t = (\mathbf{v_f} - \mathbf{v_i})/\mathbf{a}$$

$$\Delta t = (0 - 3.1)/(-1.7)$$

$$\Delta t = 1.8 \text{ s} \quad \text{(answer } \mathbf{D}\text{)}$$

Now we must add on the additional 3.0 s that she falls before she is moving at exactly 3.1 m/s, thus: $\Delta t_{total} = 1.8 + 3.0 = 4.8$ s (answer **D**)

Kinematics Problems XIX

A loonie is dropped into a wishing well and the distance traveled is $5t^2$ meters, where t is the time. Calculate the depth of the well, if the water splash is heard 3 seconds after the coin was dropped, and if the speed of sound is 335 m/s.

A. 41 m

B. 141 m

C. 200 m

D. 50 m

E. 250 m

Solution

Answer: A

Justification: For this question we are given the following:

- Distance traveled = $5t^2$
- Total time that the coin traveled = 3 s
- Speed of sound = 335 m/s

Let t_1 be the time it takes the coin to reach the bottom of the well. If D is the depth of the well, we can use the distance relationship above to write:

$$D = 5t_1^2$$

Let t_2 be the time it takes the sound wave to reach the top of the well. We know that sound travels at a constant velocity here, so we can use the following equation:

$$\text{Distance} = \text{Velocity} \times \text{Time} \quad \Rightarrow \quad D = 335 \times t_2$$

Solution 2

We can combine the two previous equations, since each is equal to D:

$5t_1^2 = 335 \times t_2 \quad => \quad t_1^2 = 67 \times t_2$

We know that it took a total of 3 s after the coin was dropped for the sound to reach the top of the well. Therefore the relationship between t_1 and t_2 is:

$$t_1 + t_2 = 3 \quad => \quad t_2 = 3 - t_1$$

We can now substitute t_2 into the previous equation:

$t_1^2 = 67 \times t_2 \quad => \quad t_1^2 = 67 \times (3 - t_1)$

We can rearrange this into a quadratic equation:

$t_1^2 + 67\, t_1 - 201 = 0$

If we solve for this quadratic equation we get two solutions, only one of which is positive (time cannot be negative):

$t_1 = 2.88$ s

Solution 3

We can now calculate the depth D of the well:

$$D = 5t_1^2 = 5(2.88)^2 = 41.37 \approx 41 \text{ m} \quad \text{(answer } \mathbf{A}\text{)}$$

Kinematics Problems XX

To design a runway of an airport, the engineers are to consider the following parameters:

a)The lowest acceleration rate of any airplane is 3 m/s^2.

b)The takeoff speed for the airplanes must be at least 65 m/s.

What is the **minimum** allowed length for the runway (to the nearest meter), assuming that the minimum acceleration is to be used?

A. 700 m

B. 705 m

C. 800 m

D. 805 m

E. 1000 m

Solution

Answer: B

Justification: For this question we are given the following:

- The airplanes always start from a stationary position, therefore we know that initial velocity $\mathbf{v_i}$ = 0 m/s
- Minimum final velocity of the plane before it leaves the runway is $\mathbf{v_f}$ = 65 m/s
- Minimum acceleration of the airplanes is $\mathbf{a}$ = 3 m/s^2

We are looking for the distance, therefore we can use the equation:

$$\boldsymbol{v_f}^2 = \boldsymbol{v_i}^2 + 2\boldsymbol{a}d$$

Rearranging, we get:

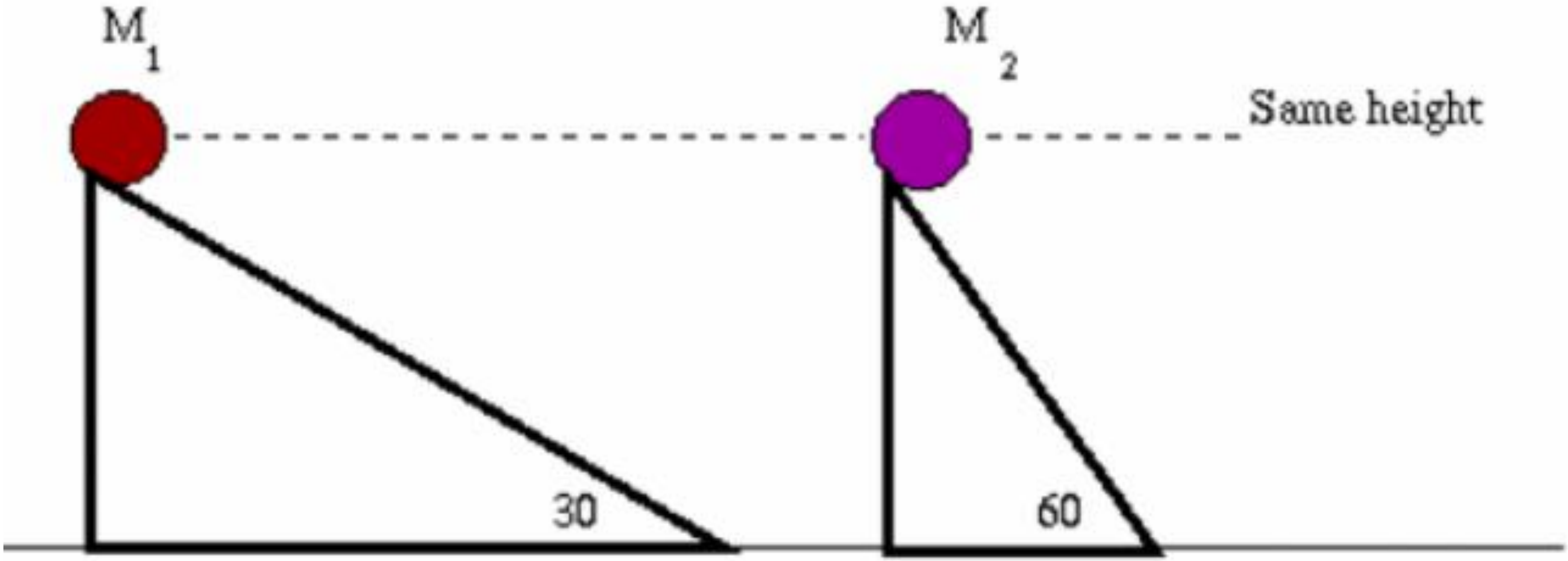
$$d = \frac{\boldsymbol{v_f}^2 - \boldsymbol{v_i}^2}{2\boldsymbol{a}} = \frac{(65)^2 - (0)^2}{2(3)} = 704.17\ m$$

Solution continued

Normally, when we round off we would round down to the nearest meter, giving us 704 m. However, in this case anything below 704.17 m will be less than the minimum and so we need to round off to 705 m. Therefore the correct answer is **B**.

Kinematics Problems XXI

Consider two frictionless inclined planes as shown. Identical balls M_1 and M_2 are released at the same time.



Kinematics Problems XXI continued

Compare the speeds of the two masses when they reach the bottoms of their respective inclines.

A. M_1 is travelling faster than M_2.

B. M_2 is travelling faster than M_1.

C. M_1 and M_2 are travelling at the same speed.

D. It is impossible to tell.

Solution

Answer: C

Justification: For this question we need to use energy conservation.

We know that since both inclines are frictionless, we know that all gravitational potential energy of the balls at the top of the inclines will be converted into kinetic energy at the bottom of the inclines.

Since both balls are at the same height, and have the same mass (they are identical), we know that they must have the same gravitational potential energy at the top of each incline ($E_P = m\mathbf{g}h$).

Therefore, by the above logic, both of the balls will have the same kinetic energy at the bottom of each incline.

Since $E_K = \frac{1}{2}m\mathbf{v}^2$, and since the mass of each ball is equal, the only way their kinetic energies could be the same is if their velocities were also equal. Therefore the balls M_1 and M_2 will be travelling at the same speed at the bottom of their respective inclines (answer **C**).

Kinematics Problems XXII

A car spends two hours driving at 20 km/h, and then an hour driving at 50 km/h. What is the average speed of the car?

A. 50 km/h

B. 45 km/h

C. 35 km/h

D. 30 km/h

E. 20 km/h

Solution

Answer: D

Justification: This answer can be answered conceptually. We know that the average speed of the car is the change in its distance divided by the total time taken. Therefore it cannot simply be an average of the two speeds (answer C), but must be a weighted average depending on the time the car spent travelling at each speed. Because the car spent spent twice as long travelling at 20 km/h compared to how long it drove for 50 km/h, the average will be weighted much closer to 20 km/h than 50km/h. This rules out options A, B and C. We also know the answer cannot be 20 km/h, because the car did travel faster than 20 km/h for part of the time. Therefore the only possible answer is **D**.

We can also do this question quantitatively. We need to add up the total distance travelled for each hour and divide by the number of hours.

Solution continued

Since the car was travelling at 20 km/h for two hours, we know that it must have travelled: 20 km/h × 2 h = 40 km

Since the car was travelling at 50 km/h for one hour, we know that it must have travelled: 50 km/h × 1 h = 50 km

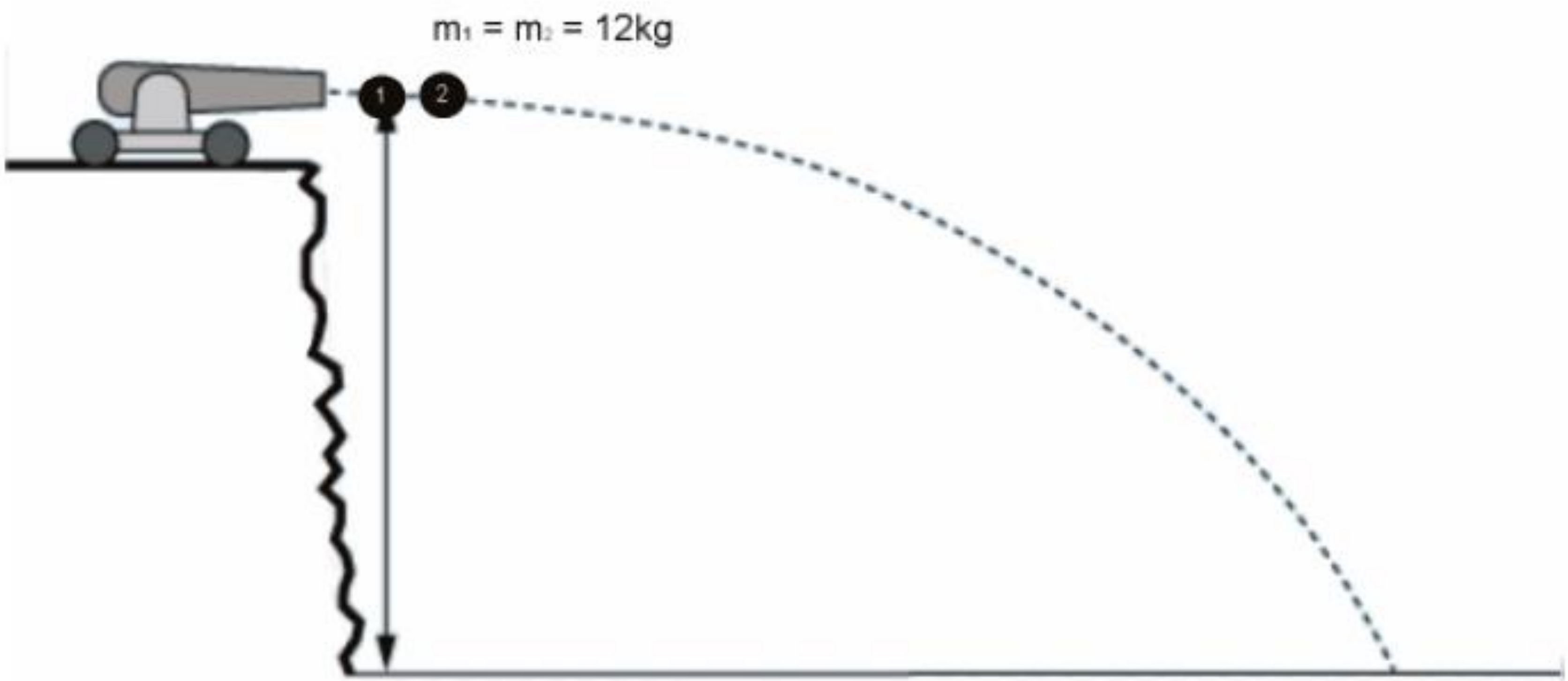
Therefore the total distance travelled was: 40 km + 50 km = 90 km

To find out the average velocity, we just divide the total distance by the total number of hours travelled (3 hours):

90 km / 3 h = 30 km/h (answer **D**)

Kinematics Problems XXIII

A cannon fires cannon ball 1 of mass m_1 = 12 kg horizontally at constant velocity $\boldsymbol{v}$ = 20 m/s. At the same time, cannon ball 2 of mass m_2 = 24 kg is dropped from an equal height. The fired ball lands after a time t_1, while the dropped ball lands after a time t_2.



Kinematics Problems XXIII

Ignoring air resistance, which of the following is true?

A. $t_1 > t_2$

B. $t_1 < t_2$

C. $t_1 = t_2$

D. It is not possible to determine the relationship between t_1 and t_2

Solution

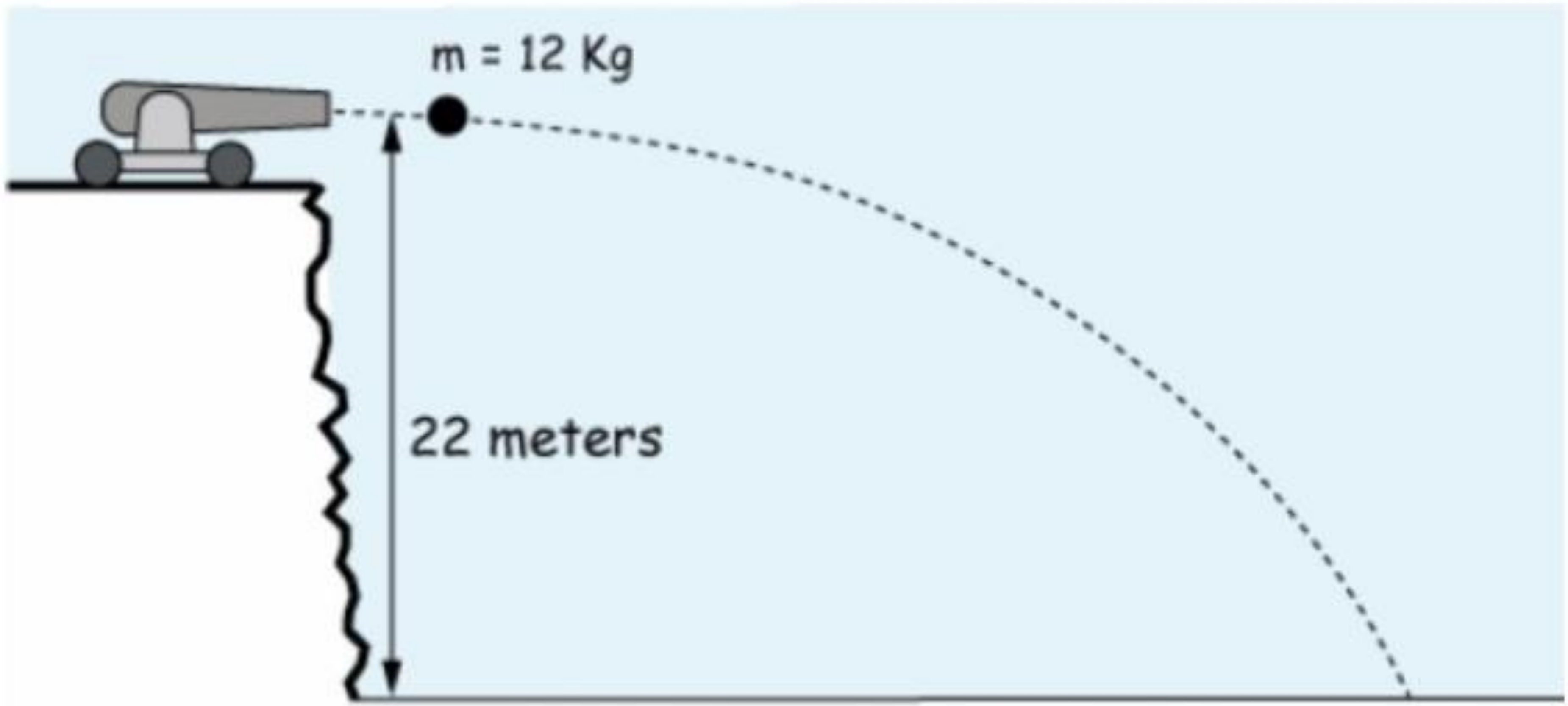
Answer: C

Justification: This answer does not require any calculations. First of all, it is important to note that both the cannon balls start with the same **vertical** velocity. Cannon ball 2 drops from rest, and therefore has a vertical velocity of zero. Cannon ball 1 is fired **horizontally**, so while it does have an initial component of velocity in the horizontal direction, it does not initially have any vertical velocity (it is also zero).

We also know that the acceleration due to gravity is the same for both balls (regardless of mass), and since the balls start with the same vertical velocity (zero) and are dropped from the same height, then they must also land at the same time (answer **C**).

Kinematics Problems XXIV

The same cannon from the previous question again fires a cannon ball of mass m = 12 kg horizontally with velocity $\boldsymbol{v}$ = 20 m/s from a height of 22 m.



Kinematics Problems XXIV continued

How far will the cannon ball travel horizontally before it lands?

Ignore air friction, and assume gravitational acceleration to be $10 m/s^2$ and downwards to be the positive direction.

A. 18 m

B. 22 m

C. 42 m

D. 98 m

Solution

Answer: C

Justification: To calculate the distance traveled, we must first find the time it took the cannon ball to reach the ground. We know from the previous question that this time is not affected by the horizontal velocity of the ball, but rather by its **initial vertical velocity** ($\boldsymbol{v_i} = 0$) and the acceleration due to gravity.

Therefore we can use the following equation:

$d = \boldsymbol{v_i}t + \frac{1}{2}\boldsymbol{a}t^2$

Substituting in our values we get:

$(22) = (0)t + \frac{1}{2}(10)t^2$

$t^2 = 4.4$

$t = 2.1$ s

Solution continued

Travelling at a constant **horizontal** velocity of 20 m/s, we can calculate how far the cannon ball would travel in 2.1 seconds:

Distance = 20 × 2.1 = 42 m (answer **C**).

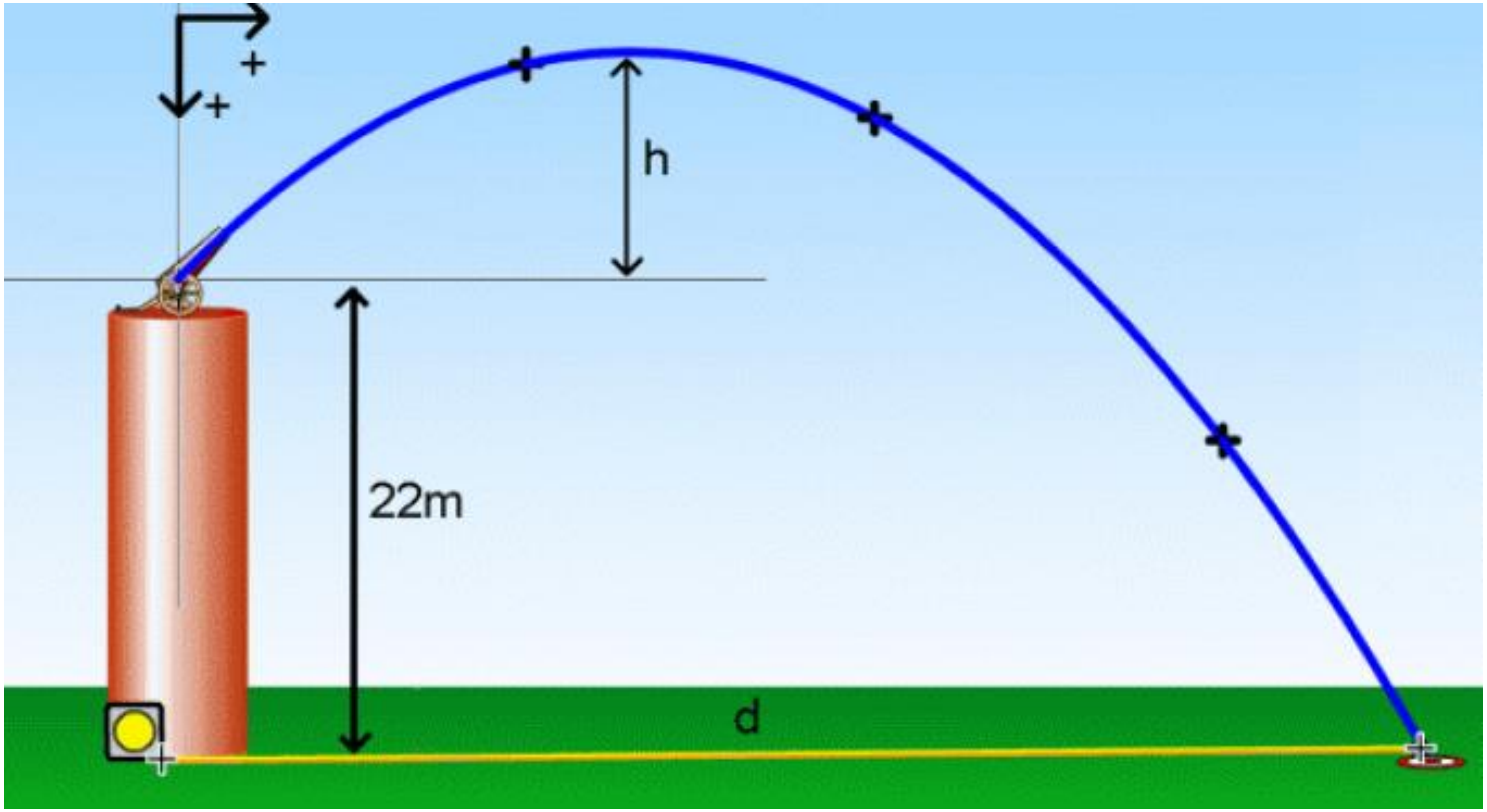
If you used the horizontal velocity (20 m/s) as $\boldsymbol{v_i}$ in the first equation, you would need to use the quadratic formula to solve this problem, and

will receive either t = 0.9 s or t = 4.9 s, depending on the sign of the value in your equation (positive initial vertical speed or negative initial vertical speed). These results lead to answers of 18 m (A) or 98 m (D).

If you used $\boldsymbol{v}$ = 20 m/s as a constant vertical velocity (downwards), the answer will be t = 1.1 s, yielding an answer of 22 m (B).

Kinematics Problems XXV

The recoil of the previous blast has caused the cannon to aim 45º up from the horizontal. The cannon again fires a cannon ball of mass *m* = 12 kg, with initial velocity of 40 m/s.



Kinematics Problems XXIII

How high above the cliff (*h*) does the cannon ball fly before it begins to fall? How long (*t*) does it take to get to the top of its arc? How far (horizontally) does the cannon ball travel before hitting the ground (*d*)?

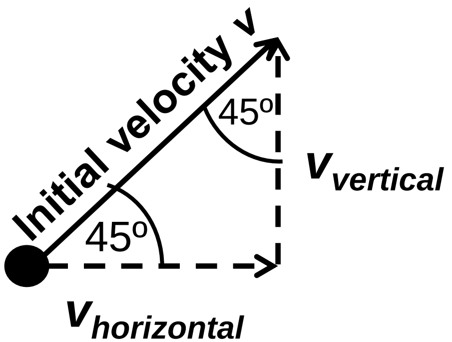
Ignore air friction, and assume gravitational acceleration to be $10 m/s^2$.

A. h = 118 m	t = 2.8 s	d = 229 m
B. h = 40 m	t = 2.8 s	d = 180 m
C. h = 80 m	t = 4.0 s	d = 340 m
D. h = 240 m	t = 4.0 s	d = 448 m

Solution

Answer: B

Justification: This problem can be thought of as a superposition of horizontal motion with constant velocity and vertical motion with acceleration due to gravity. To find the maximum height that the cannon ball reaches, we are looking only at the vertical velocity – the ball will reach its maximum height when the vertical velocity reaches zero at the top of the arc (when the ball is stationary). Therefore the first step we need to take is to calculate the vertical and horizontal components of the cannon ball's velocity.



We can use the sine rule to calculate these components:

$\boldsymbol{v_{horizontal}} = \boldsymbol{v} \times \text{Sin}\ (45^{o}) \div \text{Sin}\ (90^{o})$
$= 40 \times 0.71 \div 1 = 28.3\ \text{m/s}$

$\boldsymbol{v_{vertical}} = \boldsymbol{v} \times \text{Sin}\ (45^{o}) \div \text{Sin}\ (90^{o})$
$= 40 \times 0.71 \div 1 = 28.3\ \text{m/s}$

Solution continued

Now that we know the initial vertical velocity, the final vertical velocity at the top of the arc (zero), and the acceleration of the ball due to gravity (in this case it is – 10 m/s, since we are considering the initial upwards acceleration to be positive), we can use the following equation to find the time it takes the ball to reach its maximum height:

$\boldsymbol{v_f} = \boldsymbol{v_i} + \boldsymbol{a}t$

$0 = 28.3 + (-10)t$

$t = 2.83 \approx$ **2.8 s**

Now that we know the time we can use the following equation to calculate the maximum height that the cannon ball reaches above the cliff:

$h = \boldsymbol{v_i}t + \frac{1}{2}\boldsymbol{a}t^2 = (28.3)(2.8) + \frac{1}{2}(-10)(2.8)^2 =$ **40 m**

Note: It is important here that you use the full answers calculated, and not to use the rounded off versions.

Solution

The last thing we need to calculate is how far the ball flies horizontally before it hits the ground. We know it travels at a constant horizontal velocity of 28.3 m/s (it is constant because there is no air friction). Therefore all we need to know is how long it flies for before it hits the ground. We already know the time it takes the ball to reach its maximum height (2.8 s). Now we need to find out how long it takes to fall to the ground from this height.

Since the cliff is 22 m high, and the ball flew 40 m vertically above the cliff, we know the height of the ball at its apex above the ground is 62 m. We also know its initial velocity is zero, since at the apex it is stationary. Therefore we can use the following equation:

$d = \boldsymbol{v_i} t + \frac{1}{2}\boldsymbol{a}t^2$

$(62) = (0)(2.8) + \frac{1}{2}(10)t^2$

$t^2 = 12.4 \quad \rightarrow \quad t = \sqrt{12.4} = 3.5$ s

Solution

We can now add the two times together (2.8 + 3.5), and we know the cannon ball flew for 6.3 seconds before it hit the ground.

We can now us the constant horizontal velocity to calculate the distance it travelled:

Distance = Velocity × Time = 28.3 × 6.3 = 179.6 ≈ **180 m**

Note: Once again, it is important here that you use the full answers calculated, and not to use the rounded off versions, otherwise you will get the answer of 178 m.

The answer which has all three variables correct (h = 40 m, t = 2.8 s, and d = 180 m) is answer **B**.